I. Contact Information

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II. Visiting Scholars Grant

Grant Number: 21200-12-039547
Grant Title: “Monetary and Exchange Rate Regimes: Implications for Inflation and Growth”
Award Amount: $19,647.25

III. Accomplishments

The Visiting Scholars Grant provided an opportunity to pursue a research agenda that Professor McDermott and I had talked about with Professor Warren Weber, our Visiting Scholar, over the past several years. We are grateful for the funding and support provided by the Grant Program.

During the course of the grant, we have produced a paper that is being prepared for submission to the highly-ranked Journal of Monetary Economics. (Draft version attached). We anticipate submission by May 2015. Drafts of this paper were presented at the Brownbag series in the Economics Department at USC and at Davidson College. (Fliers attached). In addition, we have a Ph.D. student, Eun Son Lim, who is working with us. Dr. Weber will be serving as a reader on her dissertation.

Professor Weber has also been team-teaching with Professor McDermott, the newly-instituted “Research Methods” semester-long workshop for Ph.D. students. Professor Weber has coached students on their topics as well as on their presentations. Through this workshop, Eun Son Lim has completed a draft of a research paper that will be an essay for her dissertation. Professor Weber has also given two Brownbag talks in the department. (Fliers attached). These are open to graduate students and faculty.

Our research has also spawned a few other projects addressed in our grant. We will begin work on a second project that examines why countries choose different monetary regimes. Our Ph.D. student will also explore additional topics related to exchange rate regimes that were included in our grant proposal.
Finally, Professor Weber has affiliations with the Bank of Canada and Federal Reserve Bank of Atlanta. In his work on projects with these other institutions, he lists his affiliation with the University of South Carolina. This is another avenue through which the grant has provided visibility for the Economics department and the university.

IV. **Expenditure of Funds**

The expenditure of funds was as follows:

- Salary - $15,400.00
- Fringes - $1,293.60
- Travel Expenses - $2,953.65
  - March 4\textsuperscript{th} – 6\textsuperscript{th} (2 nights)
  - April 1\textsuperscript{st} – 3\textsuperscript{rd} (2 nights)
  - April 15\textsuperscript{th} – 16\textsuperscript{th} (1 night)
  - September 9\textsuperscript{th} – 11\textsuperscript{th} (2 nights)
  - October 15\textsuperscript{th} – 17\textsuperscript{th} (2 nights)
  - November 3\textsuperscript{rd} – 5\textsuperscript{th} (2 nights)
  - December 3\textsuperscript{rd} – 4\textsuperscript{th} (1 night)

V. **Attachments**

- Flier for Breuer’s Seminar at Davidson College, Nov. 6, 2014
- Flier for Breuer’s Brownbag at USC, Nov. 5, 2014
- Flier for Weber’s Brownbag at USC, Oct. 15, 2014
- Flier for Weber’s Brownbag at USC, April 16, 2014
- Manuscript Draft
RISK MANAGEMENT AND RURAL TO URBAN MIGRATION IN INDONESIA
HILLARY CARUTHERS
ASSISTANT PROFESSOR OF ECONOMICS, LAWRENCE UNIVERSITY
THURSDAY, OCT. 2 • 4:30 P.M. • CHAMBERS 3155

THE IMPACT OF THE MEDICAID POLICY ENVIRONMENT ON PRENATAL CARE UTILIZATION AND BIRTH OUTCOMES AMONG IMMIGRANTS
 TIFFANY GREEN
ASSISTANT PROFESSOR OF HEALTHCARE POLICY & RESEARCH, VIRGINIA COMMONWEALTH UNIVERSITY
THURSDAY, OCT. 16 • 4:30 P.M. • CHAMBERS 3155

INTERNATIONAL EVIDENCE ON THE MONEY-INFLATION RELATIONSHIP
JANICE BOUCHER BREUER
PROFESSOR OF ECONOMICS, MOORE SCHOOL OF BUSINESS, UNIVERSITY OF SOUTH CAROLINA
THURSDAY, NOV. 6 • 4:30 P.M. • CHAMBERS 3155

THE DEPARTMENT OF ECONOMICS IS NOW ON FACEBOOK.
ECONOMICS Seminar:

Jan Breuer
University of South Carolina

“International Evidence on the Money-Inflation Relationship”

Wednesday, November 5th
12:30 PM
Room 119
ECONOMICS Seminar:

Warren Weber
University of South Carolina

“Means of Payment”

Wednesday, October 15th
12:30 PM
Room 119
ECONOMICS Seminar:

Warren Weber
University of South Carolina

“The National Bank Note Puzzle”

Wednesday, April 16th
12:30 PM
Room 402
International Evidence on the Money-Inflation Relationship

Jan Breuer, John McDermott, and Warren Weber

January 16, 2015

Abstract

We conduct an empirical study of the money-inflation relationship using postwar data for 125 countries using four different levels of time aggregation: long run, decade, half-decade, and annual. We find support for the classical quantity theory of money when country money growth is high and sustained over many years regardless of the degree of time aggregation. When it is low – no matter what the degree of time aggregation – inflation responds to changes in money growth with an elasticity much smaller than unity. Further, we find that for low money growth countries, the elasticity gets smaller as the degree of time aggregation decreases. We suspect that inflation targeting may introduce negative bias in our estimates of the money-inflation relationship and that the bias worsens as the degree of time aggregation
declines. We investigate with simulations and find that the size of the bias depends on the time series properties of money growth.

1 Introduction

The quantity theory of money has considerable appeal. It is simple and logical and assigns a direct chain of causality from the amount of money in circulation to the price level. The quantity theory has served as a quick guide to policy through the ages and is the starting point for more sophisticated theories of money and economic activity.\footnote{See, for example, the work of Sidrauski (1967) on money and growth.} In the Classical version, the causality between money growth and inflation is theorized to be one-for-one. It has been studied empirically, on and off, for decades, and a consensus has developed around the following observation: the Classical version seems to hold best during periods of high money growth. Therefore, the first major purpose of this study is to examine this claim by looking at the extent to which the empirical money-growth relationship is driven by the presence of high money growth countries.

We concentrate on the coefficient of money growth in a regression with inflation as the dependent variable. The classical quantity theory predicts that this coefficient is unity, but is silent over whether this relationship should emerge only in the long run or whether at shorter horizons, too. Accordingly, a second purpose of the paper is to investigate the effect of time aggregation...
on the results. We begin by creating a cross-section of countries using annual data averaged over the sample period available for each country – our “long run” data set. The time span coverage for each country varies quite a bit: our earliest date is 1951 and the latest is 2011, but many countries have reliable data for only a small subset (fewer than 20 observations) of this sample. Given data availability and other data considerations, we have a sample of 125 countries. We then construct panel data sets by averaging the annual data for each country by decade and by half-decade; we also use the annual data to create a panel data set that has much less time aggregation than our three other data sets. The question of which monetary aggregate to use comes up in studies like ours. Therefore, a third purpose of our study is to investigate whether our results are dependent on the monetary aggregate. Because money and price data reported to the IMF share some inconsistencies, we offer a thorough discussion of the data and any inconsistencies.

Using these four data sets for two different measures of money, we estimate the money growth-inflation relationship using panel techniques. We find that regardless of the level of time aggregation or the money measure, the quantity theory receives considerable support. However, when we divide the data into two groups based on whether a country’s average money growth is above or below 20 percent, we find that the coefficient on money is close to or moderately above 1.0 only for the group with high money growth. For those with low money growth, the coefficient is much smaller, but is generally significant. These results hold irrespective of the money measure used.
Inflation targeting may be responsible for coefficient estimates that are biased below unity, particularly if inflation-targeters maintain a low rate of money growth. This is because inflation targeting creates simultaneous equations bias between money and inflation. Since inflation targeters adjust money downward when inflation rises (and vice-versa), the bias on the estimated money-inflation relationship may be downward.\(^2\) Since our results show that the bias gets weaker with a greater degree of time averaging, we conduct simulations to examine the influence of simultaneous equations bias present in the annual data and time aggregation on the estimated money-inflation relationship. We first establish that simultaneity between money and inflation will produce biased estimates in our regression of inflation on money growth. We then examine whether the bias changes with the degree of time aggregation. We find that it does not. However, we argue that simultaneous equations bias will be stronger if money growth is autocorrelated and that the extent of bias depends on the characteristics of the autocorrelation in money growth.

The paper is organized as follows. Section 2 presents the basic theory and describes existing literature. In Section 3 we discuss the data in detail. Section 4 contains our long-run results using the sample-averaged cross-section of data and for both measures of money. Section 5 provides results at three

\(^2\)A different story may hold for the high money growth countries. For these countries, central banks may ratchet up the money supply as inflation rises in order to maintain real balances. This may create positive simultaneous equations bias on the estimated money-inflation.
different levels of time aggregation: decade, half-decade, and annual. Section 6 reports evidence that inflation-targeters were able to achieve lower rates of money growth. Our results and Section 7 investigates potential sources of bias in the empirical money-inflation relationship.8 concludes.

2 Theory and Literature

The quantity theory is a very old idea, dating back to the 19th Century, but popularized by Irving Fisher (1911). As Milton Friedman (1987) explains, the quantity theory began as an identity – the “quantity equation” – to relate the circulation of money to the number of transactions in the economy. This identity was written $MV = PT$, where $M$ is the nominal stock of money, $V$ is velocity, $P$ is the level of prices, and $T$ is the number of transactions that occur annually in the economy. It is difficult to measure the number of transactions, so it has become customary to proxy $T$ with a monotonic function of GDP, $Y$. Accordingly, the quantity equation can be written as $P = MV/Y^\delta$ where we assume $T = Y^\delta$ and the parameter $\delta > 0$. It is often assumed that $\delta = 1$. The quantity equation has been used to relate inflation and money growth by assuming the $V$ is a constant and uncorrelated with $M$ or $Y$. Let lower case letters refer to natural logs. Then in rate of change form, the quantity theory is:

$$\Delta p = \Delta m - \delta \Delta y \quad (1)$$
where $\Delta p$ is the rate of inflation, $\Delta m$ is the growth rate of money, and $\Delta y$ is the growth rate of real GDP. In this form, the quantity theory of money says that the inflation rate moves one-for-one with money growth controlling for output growth. In terms of a rule of thumb, the quantity theory says that a country’s inflation rate will be the difference between the rate of growth of money and output. This heuristic is at the core of current monetary policy debate about the long-run effects of quantitative easing.3

Tests of the quantity theory have been carried out by McCandless and Weber (1995), Lucas (1996), Frain (2004), and DeGrauwe and Polan (2005).4 All of these studies examine the quantity theory with a large cross-section of countries using data averaged over the sample period for which data is available. Country coverage in these studies ranges from about 80-160 countries. Several of the studies consider country sub-samples. Teles and Uhlig (2013) is based on a developed countries only. Whatever the case, empirical support for the quantity theory is consistently produced from the cross-sectional studies. Two of the studies – Frain (2004), and DeGrauwe and Polan (2005) – split the sample into high and low-inflation groups and only find evidence favorable to the quantity theory for the high-inflation countries. Further, DeGrauwe and Polan (2005) tests the quantity theory using panels of data.

3The theory in its modern form has been recast in terms of money demand and money supply. The foundational work was done by Baumol (1952); Tobin (1956); and Miller and Orr (1966). More recent studies have used the theory to comment on the stability of money demand and the welfare effects of inflation. See, for example, Lucas (2000), Ireland (2009), and Lucas and Nicolini (2013).

4Other papers include Vogel 1974, Barro 1990, and ... (see DeG & Polan Table)
averaged at intervals of 1 - 6 years. He finds blah blah. If one result stands out in this literature, it is that the quantity theory works very well at explaining inflation in countries that have a history of high money growth. It is less clear how well it holds for countries that have low rates of money growth (and inflation). Sargent and Surico (2011) explore this idea for the United States and Teles and Uhlig (2013) for the OECD countries. Sargent and Surico (2011) propose that different monetary rules may account for systematic differences in the money-inflation relationship in the United States over time.

One difficulty in testing equations like (1), especially for low-inflation countries, is the treatment of the change in velocity $\Delta v$, since we have no way to observe velocity directly. Velocity’s change is either related to the nominal interest rate or included in the error term. In this paper, we follow the latter strategy – as do McCandless and Weber (1995), Frain (2004), and DeGrauwe and Polan (2005). To be valid, we must assume that velocity is orthogonal to the variables that we can measure and that $\Delta m$ and $\Delta y$ are exogenous.

Even in low-inflation economies, though, the quantity theory has found support. Lucas (1980) thought the US fit the theory, and Teles and Uhlig (2013) show that, between 1970 and 1990, in their sample of low-inflation, OECD countries the theory is supported. In the second subsample, 1990 - 2005, however, it does not work nearly as well, which they attribute to inflation targeting. Because interest was near zero after 2005, they end the
3 Data

Our primary data on money and prices comes from the International Monetary Fund, *International Financial Statistics* (CD-ROM, 2014). The price level \( p \) is measured by the natural log of the *CPI*. Data on the consumer price index is the most commonly used measure of inflation in the literature. For most countries, the time series coverage of the *CPI* is consistent, widely-reported, and contains no gap years though the initial starting year for collection of the data varies widely. A gap occurs if, e.g. the *CPI* is reported from 1970 - 2011 but is missing data for, say, 1992 and 1993. A gap in the *CPI* of one year produces a two-year gap in the inflation rate data. Gaps in the *CPI* data occur for five countries in our sample. \(^5\)

The money data is more problematic. First, standardization of definitions of money is less consistent than may be commonly supposed. There are various versions of “broader” money like *M2* or *MQM* (money plus quasi-money) for example. The same is true for a narrower measure of money. Some countries report *M1* whereas others report *Money*. This is probably due to the different institutional arrangements in central banking around the world and specific data collection algorithms used by countries to gather data.

\(^5\)The countries with gaps in the CPI data are: The Republic of Congo (1997), Djibouti (1988-1999), Iraq (1979-1989), Lesotho (1997-1998), and Rwanda (1994). Also, the United Kingdom reports data on the *CPI* beginning in 1988. Prior to then, the producer price index was reported.
data. The fact remains that not all countries report data on $M2$; some report $MQM$, and some report both.\footnote{The same is true for $M1$ and $Money$.} These two measures of money are broadly comparable and where countries report both, the correlation between the series is 0.72. Neither series, however, is available for as many countries as is the $CPI$ data. In general, $MQM$ is a longer time series than $M2$ but typically ends in 2008. While $M2$ coverage is shorter (having a later start year), it generally includes the year 2010 (and beyond) and therefore has the advantage of including the financial crises period. We therefore chose our measure of money based on an algorithm that balanced length of series (number of observations) and its latest date. We select $M2$ only if it ended at least 3 years later than $MQM$ and had no more than 10 fewer observations than $MQM$. As with prices, we converted money to natural log form to obtain $m$. We apply the same algorithm to $M1$ and $Money$. In addition to differences in the money measures reported by each country, the sample period coverage varies, too. As with the $CPI$ data, not all countries share the same beginning and ending dates. And, seven countries in our data set contain gaps in $m$.\footnote{Gaps in $m$ occur for Congo, Dem. Republic (1996-1999), Iran (1984-1985), Iraq (1977-2003), Kuwait (1990), Luxembourg (1993, 1995), Mauritania (1992-2004), and Zambia (1992).}

We use real output data from the Penn World Table v. 8.0. That data now reports three distinct real output series. Each is expressed in real U.S. dollars and are comparable across time and countries. One series, $RGDP^{NA}$,
is similar to real output in earlier versions of the PWT data.\(^8\) It relies on National Accounts data, and seems to be close to \(RGDPL\) in the early versions. The series \(RGDP^O\), is based on national production, while \(RGDP^E\), is based on national expenditure. When the terms of trade are very different from unity, these measures can be diverge substantially. We use the natural log of \(RGDP^{NA}\) for our measure of output \(y\). We do so mainly because it allows comparison with earlier studies.

The Data Appendix reports summary information on the sample period where \(p\), \(m\), and \(y\) are jointly available. The table also indicates whether, based on our algorithm, \(M2\) or \(MQM\) was selected. Due to the large gaps in data for Iraq and Mauritania, we drop these two countries from the analysis. In all, we have consistent data on \((p, m, y)\) for 125 countries.

4 Results for High and Low Money Growth Countries

The strong relationship between money growth and inflation that appears using long-run cross country averages is shown in Figure 1 in which we plot inflation rates against M2 money growth for the 125 countries in our sample of countries for which we have data on M2.\(^9\) The correlation between money

\(^8\)In PWT 8, output is reported in \textit{level} terms. Previously, output was reported in per capita terms.

\(^9\)Graphical analyses like this have appeared in the literature over the years, notably by Vogel (1974), Lucas (1980), and McCandless and Weber (1995)
growth and inflation is 0.97.\footnote{We also computed the correlation between “corrected money growth,” $\Delta m - \Delta y$, and inflation. It is 0.98. Further, a plot of inflation against corrected money growth looks almost identical to Figure ???. For that reason we do not include the plot here.}

Figure 1: Sample-average Inflation and M2 Money Growth by Country

The same strong relationship between money growth and inflation also holds when the monetary aggregate is M1 as is shown in Figure 2 where we plot inflation and money growth using M1 as the measure of money. The correlation is only slightly lower at 0.95. Our sample contains 123 countries in this case.

The dotted line in both figures is the 45° line drawn through the sample
Figure 2: Sample-average Inflation and M1 Money Growth by Country
averages. The points in both figures lie close to the dotted line indicating that there is a linear relationship between the two variables with a slope equal to one. To check whether this is the case, we run the regression equation

$$\Delta p_j = \alpha + \beta \Delta m_j + \delta \Delta y_j + \varepsilon_j,$$

where lower case letters refer to natural logs and $\Delta p_j$ is the rate of inflation in country $j$, $\Delta m_j$ is the rate of money growth in country $j$, $\Delta y_j$ is the rate of real GDP growth in country $j$, and $\varepsilon_j$ is an iid distributed random variable with mean 0 and variance $\sigma^2$. We include $\Delta y$ in the regression equation to account for the fact that a country’s long run rate of output growth might influence the long run rate of inflation that it experiences.

We estimate equation (2) by OLS using data averaged over the sample period for which $(\Delta p, \Delta m, \Delta y)$ is jointly available. This insures that the coefficient estimates are derived from correlations amongst variables that have been constructed over the same sample period. This is an important point to note about how we average the data. The averages we use to estimate equation (2) are not computed over the raw data available for each individual series for a country. Had we used this approach, it is possible that for country $j$, $\Delta p$ would be averaged over a longer or shorter sample period, say than $\Delta m$ or $\Delta y$ if each of the series have different starting and ending years.\footnote{The same rule for the sample period over which averaging is done was used for the figures above.}
Table 1: Inflation and M2 Money Growth: Country Averages

<table>
<thead>
<tr>
<th>Sample Restrictions</th>
<th>$\beta$</th>
<th>95% Confidence Interval</th>
<th>Obs</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.97***</td>
<td>0.93 - 1.02</td>
<td>125</td>
<td>0.96</td>
</tr>
<tr>
<td>for $\Delta m_j &lt; 0.20$</td>
<td>0.82***</td>
<td>0.65 - 0.98</td>
<td>86</td>
<td>0.57</td>
</tr>
<tr>
<td>for $\Delta m_j \geq 0.20$</td>
<td>1.04***</td>
<td>0.94 - 1.13</td>
<td>39</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Note: Country coverage includes countries with at least 10 annual observations.

4.1 Results with M2

The results using M2 as the monetary aggregate are shown in Table 1. The results using all 125 observations shown in Row 1. The estimate of $\beta$ is very close to 1.0 and the 95 percent confidence interval contains 1.0, but does not include 0.0. Moreover, 96 percent of the variation in inflation across countries is explained by money growth and output growth. This result suggests that over the long run a country can expect to experience a rate of inflation equal to the rate of money growth it runs over that same long run period.

A striking feature of Figure 1, however, is the cloud of countries with low rates of money growth. To see what is going on in this cloud, we divide our sample into two categories. The first is countries with M2 money growth rates less than 0.2; that is, with $\Delta m_j < 0.20$. We designate these countries as “low money growth countries.” There are 86 such countries in our sample. We designate the remaining 39 countries with $\Delta m_j \geq 0.20$ as “high money growth countries.”
We plot inflation against M2 money growth for the low money growth countries in Figure 3. The points in this figure do not cluster along a line; the correlation is much lower at 0.64. Further, to the extent that the points fall along a line, it appears questionable whether that line is the 45° line, which suggests that the long run relationship between inflation and money growth may be different for low money growth countries and high money growth countries.

Figure 3: Sample-average Inflation and M2 Money Growth, Low Money Growth Countries

To check whether this is the case, we rerun equation (2) with the sample restricted to those countries with relatively low money growth. The results
are given in Table 1. The estimate of $\beta$ for low money growth countries, shown in Row 2, falls to 0.82 from 0.97. Further, for this sample of countries, the 95 percent confidence interval no longer contains 1.0, although it still does not include 0.0. The contrast between the low money growth countries and the high money growth countries is seen by comparing the results in Row 2 with those in Row 3, where we display the results of estimating equation (2) for the sample of high money growth countries. For these countries, $\hat{\beta} = 1.04$, higher than the estimate of $\beta$ in the full sample. Further, the 95 percent confidence interval contains 1.0.

Taken as whole, we interpret these results as supporting the claim that the strong correlation between inflation and M2 money growth is largely due to the presence in the sample of countries with high average money growth rates. To obtain more evidence, we rerun equation (2) dropping the countries with largest money growth rates one at a time until the 95% confidence interval no longer includes 1.0. We find once we delete the four countries with the largest long run average growth rates – countries with long run average growth rates $> 0.55$ – that the 95% confidence interval no longer includes 1.0.\textsuperscript{12} For this sample, $\hat{\beta} = 0.92$.

Another way of interpreting the results is that if a country adopts a monetary regime that delivers high money growth (in our sense of $\Delta m_j \geq 0.20$), then it can experience its long run average inflation rate to be the same as the rate of money growth. However, if it chooses to adopt a monetary

\textsuperscript{12}The four countries are Angola, Argentina, Brazil, and Congo.
regime that delivers low money growth (in our sense of $\Delta m_j < 0.20$), then it can expect its long run rate of inflation to be only approximately 80 percent as high as its money growth rate.

### 4.2 Results with M1

To determine whether the results found using M2 as the measure of money are robust to the choice of monetary aggregate, we rerun the above regressions with M1 as the monetary aggregate. The results are given in Table 2.

Comparing the results using M1 with those using M2 as the monetary aggregate, we find:

1. for the entire sample of 123 countries, the estimate of $\beta$ is larger using M1 than with M2
2. for the sample of low money growth countries, the estimate of $\beta$ is also larger using M1 than with M2 and the 95 percent confidence interval now includes 1, and
3. for the sample of high money growth countries, the estimate of $\beta$ is also larger with M1 than with M2 and significantly greater than 1

Thus, it does not appear that the regression results are driven as much by the presence of high money growth countries when the monetary aggregate is M1 rather than M2, although the estimates of $\beta$ continue to be lower when high money growth countries are excluded from the regression analysis.
Table 2: Inflation and M1 Money Growth: Country Averages

<table>
<thead>
<tr>
<th>Sample Restrictions</th>
<th>$\beta$</th>
<th>95% Confidence Interval</th>
<th>Obs</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>1.02***</td>
<td>0.92 - 1.13</td>
<td>123</td>
<td>0.92</td>
</tr>
<tr>
<td>for $\Delta m_j &lt; 0.20$</td>
<td>0.84***</td>
<td>0.59 - 1.09</td>
<td>89</td>
<td>0.51</td>
</tr>
<tr>
<td>for $\Delta m_j &gt;= 0.20$</td>
<td>1.15***</td>
<td>1.02 - 1.28</td>
<td>34</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Note: Country coverage includes countries with at least 10 annual observations. $m = M1$ or $MNY$.

5 Results for Different Time Aggregation

The previous section demonstrated that the strength of the empirical evidence in favor of the quantity equation depends upon the monetary aggregate used in the analysis and the sample of countries included in the analysis. In this section, we examine the extent to which the strength of the empirical evidence in favor of the quantity equation depends on the amount of time aggregation done in the analysis. Our estimating equation becomes:

$$\Delta p_{jt} = \alpha + \beta \Delta m_{jt} + \delta \Delta y_{jt} + \varepsilon_{jt}$$

where $j$ indexes the country and $t$ indexes the time period: decade, half-decade, or annual.

To introduce the time dimension, we can average the data over any interval of time. We chose to average by calendar decade and by calendar half-decade. We also use an annual interval, too. In constructing these decade and half-decade averages, we require that at least 80 percent of the annual
observations over the decade or half-decade be available for constructing the
decade (or half-decade) averages of $(\Delta p, \Delta m, \Delta y)$. We do this because we
want to be sure that the decade and half-decade averages best capture the
time-averaged outcomes for each variable. Without this requirement, it is
possible that a single annual observation on $(\Delta p, \Delta m, \Delta y)$ within a decade
or half-decade would be calculated as the decade or half-decade average, cre-
at ing artificial variation in the averaged data that would not be present had
we had the full set of observations over the decade.

Further, we again explore the extent to which the results we obtain are
affected by the presence of low money growth observations in the sample. To
do this, once again we split the sample according to money growth rates –
again, by whether $\Delta m \geq .20$.

There are, in fact, two ways to split the sample. The first is to split
by observation: a decade or half-decade observation with $\Delta m_{j,t} < .20$ is
classified as a low money growth episode regardless of country. Similarly, a
decade or half-decade observation with $\Delta m_{j,t} \geq .20$ is classified as a high
money growth episode regardless of country. In other words, this way of
constructing the sample classifies an observation as either high or low money
growth irrespective of country or time. Thus, it is possible that a country
could have some decades that were classified as low money growth and some
that were classified as high money growth.

The second way to split the sample is by country. In this method, only
countries whose average money growth over the entire sample period is less
than 0.20, $\Delta m_{ave,j} < 0.20$, are included in the sample of low money growth decades or half-decades. This second way of splitting the sample amounts to grouping the countries into two classes, low money growth countries and high money growth countries, and then looking at the results with decade or half-decade observations for each class of such countries. This second method of restricting the same creates a more homogeneous population of country observations in the two sub-samples used in the estimation.

5.1 Results with M2

The regression results using M2 as the monetary aggregate for the decade data and the half-decade date are shown in Tables 3 and 4, respectively. In the upper panel of each table, we report pooled OLS results, where the test statistics are calculated using robust standard errors. In the lower panel, we use fixed effects estimators and time dummies. Fixed effects estimation allows for the possibility that we may be omitting country-specific unobservable factors that are correlated with our variable of interest and which may bias our estimates of $\beta$.

If we do not restrict the sample (Row 1), the story using either decadal or half-decadal data is very similar. We fail to reject the hypothesis that $\beta = 1$. The point estimate is around $\beta = .96$ and the confidence intervals all include 1.0. When we restrict the sample according to money growth – again, by whether $\Delta m \geq .20$ – we see some differences. There are, in fact, two ways to restrict the sample. First, we restrict by observation: only country-decade
Table 3: Inflation and M2 Money Growth: Panel Results by Decade Averages

### Pooled Results

\[ \Delta p_{j,t} = \alpha + \beta \Delta m_{j,t} + \delta \Delta y_{j,t} + \varepsilon_{j,t} \]

<table>
<thead>
<tr>
<th>Sample Restrictions</th>
<th>( \beta )</th>
<th>95% Confidence Interval</th>
<th>Obs (countries)</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.95***</td>
<td>0.90 - 1.01</td>
<td>441 (124)</td>
<td>0.93</td>
</tr>
<tr>
<td>for ( \Delta m_{j,t} &lt; 0.20 )</td>
<td>0.56***</td>
<td>0.40 - 0.72</td>
<td>331 (108)</td>
<td>0.33</td>
</tr>
<tr>
<td>for ( \Delta m_{j,t} \geq 0.20 )</td>
<td>0.98***</td>
<td>0.90 - 1.07</td>
<td>112 (59)</td>
<td>0.95</td>
</tr>
<tr>
<td>for ( \Delta m_{ave,j} &lt; 0.20 )</td>
<td>0.64***</td>
<td>0.51 - 0.78</td>
<td>322 (85)</td>
<td>0.44</td>
</tr>
<tr>
<td>for ( \Delta m_{ave,j} \geq 0.20 )</td>
<td>0.98***</td>
<td>0.90 - 1.05</td>
<td>119 (39)</td>
<td>0.96</td>
</tr>
</tbody>
</table>

### Panel Fixed Effects Results

\[ \Delta p_{j,t} = \alpha_i + \beta \Delta m_{j,t} + \delta \Delta y_{j,t} + \sum \gamma_t decdum_t + \varepsilon_{j,t} \]

<table>
<thead>
<tr>
<th>Sample Restrictions</th>
<th>( \beta )</th>
<th>95% Confidence Interval</th>
<th>Obs (countries)</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.95***</td>
<td>0.89 - 1.00</td>
<td>441 (124)</td>
<td>0.91</td>
</tr>
<tr>
<td>for ( \Delta m_{j,t} &lt; 0.20 )</td>
<td>0.34***</td>
<td>0.11 - 0.58</td>
<td>331 (108)</td>
<td>0.45</td>
</tr>
<tr>
<td>for ( \Delta m_{j,t} \geq 0.20 )</td>
<td>1.01***</td>
<td>0.91 - 1.11</td>
<td>112 (59)</td>
<td>0.93</td>
</tr>
<tr>
<td>for ( \Delta m_{ave,j} &lt; 0.20 )</td>
<td>0.45***</td>
<td>0.24 - 0.66</td>
<td>322 (85)</td>
<td>0.49</td>
</tr>
<tr>
<td>for ( \Delta m_{ave,j} \geq 0.20 )</td>
<td>0.98***</td>
<td>0.92 - 1.03</td>
<td>119 (39)</td>
<td>0.96</td>
</tr>
</tbody>
</table>

NOTE: Fixed Effects and decadal dummies included in panel estimation.
Decadals averages constructed when there are at least 8 annual observations per decade.
Table 4: Inflation and M2 Money Growth: Panel Results by Half-Decade Averages

<table>
<thead>
<tr>
<th>Sample Restrictions</th>
<th>β</th>
<th>95% Confidence Interval</th>
<th>Obs (countries)</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.97***</td>
<td>0.91 - 1.02</td>
<td>914 (125)</td>
<td>0.92</td>
</tr>
<tr>
<td>for Δm_{j,t} &lt; 0.20</td>
<td>0.44***</td>
<td>0.29 - 0.59</td>
<td>679 (111)</td>
<td>0.24</td>
</tr>
<tr>
<td>for Δm_{j,t} &gt;= 0.20</td>
<td>1.02***</td>
<td>0.95 - 1.08</td>
<td>235 (82)</td>
<td>0.95</td>
</tr>
<tr>
<td>for Δm_{ave,j} &lt; 0.20</td>
<td>0.51***</td>
<td>0.40 - 0.62</td>
<td>669 (86)</td>
<td>0.35</td>
</tr>
<tr>
<td>for Δm_{ave,j} &gt;= 0.20</td>
<td>0.99***</td>
<td>0.93 - 1.06</td>
<td>245 (39)</td>
<td>0.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample Restrictions</th>
<th>β</th>
<th>95% Confidence Interval</th>
<th>Obs (countries)</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.95***</td>
<td>0.90 - 1.00</td>
<td>914 (125)</td>
<td>0.90</td>
</tr>
<tr>
<td>for Δm_{j,t} &lt; 0.20</td>
<td>0.23</td>
<td>-0.01 - 0.46</td>
<td>679 (111)</td>
<td>0.38</td>
</tr>
<tr>
<td>for Δm_{j,t} &gt;= 0.20</td>
<td>0.98***</td>
<td>0.91 - 1.04</td>
<td>235 (82)</td>
<td>0.94</td>
</tr>
<tr>
<td>for Δm_{ave,j} &lt; 0.20</td>
<td>0.31**</td>
<td>0.19 - 0.43</td>
<td>669 (86)</td>
<td>0.44</td>
</tr>
<tr>
<td>for Δm_{ave,j} &gt;= 0.20</td>
<td>0.97***</td>
<td>0.91 - 1.03</td>
<td>245 (39)</td>
<td>0.95</td>
</tr>
</tbody>
</table>

NOTE: Standard errors clustered by country. Fixed Effects and half-decadal dummies included. Half-decadal averages constructed when there are at least 4 annual observations per half-decade.
or country-half-decade observations with $\Delta m_{j,t} < .20$ are included in the samples of Row 2 of each panel. In Row 3, the sample contains observations with $\Delta m_{j,t} > .20$. The second way to restrict the sample is by country. In this method, only countries whose average money growth over the sample period, $\Delta m_{\text{ave},j}$ are included. These results are in Rows 4 and 5. The second restriction amounts to grouping the data into two sets: high money growth countries and low money growth countries. The former restriction instead classifies any observation (irrespective of country or time) as either high or low money growth. This method creates a more heterogeneous population of observations in the two sub-samples used in the estimation.

The decade data in Table 3 shows a clear pattern. First, the coefficient $\beta$ on $\Delta m$ is lower in the sample with low money growth compared to that in the high-money-growth sample. This holds for in all cases. While we cannot reject $\beta = 1$ for the sample with $\Delta m_{j,t} > .20$ or $\Delta m_{\text{ave},j} > .20$, we do reject for the samples confined to low money growth. This supports the conventional view that the quantity theory only holds in countries with high inflation. Interestingly, however, for the low-growth samples we can reject $\beta = 0$ at the 1 to 5 percent level of significance. Another pattern is this: the estimates for the within-effects $\beta'$s are lower for the low-money-growth sample, but not for the high-money-growth sample.

The pattern of results using the half-decade data – see Table 4 – are

---

Recall from Section 3 that the sample period used to construct the average for any of the variables is the sample period for which the complete set ($\Delta p$, $\Delta m$, $\Delta y$) is available.
quite similar. We cannot reject $\beta = 1$ for the whole sample or for the sample defined by high money growth — whether for the pooled OLS or within-effects methods and whether we apply the criteria using $\Delta m_{j,t}$ or $\Delta m_{ave,j}$. For the low-money-growth sample, $\beta$ is lower and significantly different from zero, except in one case.

Finally, we repeat this exercise using the annual data. Because the data represents the shortest time aggregation of the sets we have constructed, we expect that the quantity-theory is less likely to be supported. We think this because we expect the $Var(\Delta m_{j,t})$ will be higher with less time aggregation and that $(Cov(\Delta p_{j,t}, \Delta m_{j,t}))$ will be smaller. This may yield smaller estimates of $\beta$ with a wider confidence interval rendering $\beta$ insignificant. Further, when using annual data, $\beta$ is an estimate that gives equal weight to any observation on $(\Delta p_{j,t} \Delta m_{j,t})$ whereas with a pure cross-section as in Table 1, equal weight is given to each observational unit — in this case, country — by construction. Since we have 125 countries total and $j = 39$ have $m_{ave} >= 0.20$, the high-inflation countries have about a one-third weight in the estimation of $\beta$ when we impose no restrictions on money growth. When we use annual data, there are three issues that matter to the estimate of $\beta$ in the unrestricted sample (Row 1) compared to the cross-section: (i) because there is no time-averaging, the data are likely to exhibit greater variation than in the cross-sectional data; (ii) the high money growth countries (which generally don’t reject $\beta = 1$) receive about a 5 percentage point less weighting in the construction of $\beta$; and (iii) simultaneity bias may be stronger. Results with the annual data are
Table 5: Inflation and M2 Money Growth: Panel Results with Annual Data

### Pooled Results

\[ \Delta p_{j,t} = \alpha + \beta \Delta m_{j,t} + \delta \Delta y_{j,t} + \varepsilon_{j,t} \]

<table>
<thead>
<tr>
<th>Sample Restrictions</th>
<th>$\beta$</th>
<th>95% Confidence Interval</th>
<th>Obs (countries)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.87***</td>
<td>0.77 - 0.94</td>
<td>4766 (125)</td>
<td>0.76</td>
</tr>
<tr>
<td>for $\Delta m_{j,t} &lt; 0.20$</td>
<td>0.07</td>
<td>-0.02 - 0.16</td>
<td>3385 (125)</td>
<td>0.02</td>
</tr>
<tr>
<td>for $\Delta m_{j,t} &gt;= 0.20$</td>
<td>1.01***</td>
<td>0.94 - 1.09</td>
<td>1381 (111)</td>
<td>0.84</td>
</tr>
<tr>
<td>for $\Delta m_{ave,j} &lt; 0.20$</td>
<td>0.25***</td>
<td>0.21 - 0.29</td>
<td>3480 (86)</td>
<td>0.13</td>
</tr>
<tr>
<td>for $\Delta m_{ave,j} &gt;= 0.20$</td>
<td>0.93***</td>
<td>0.85 - 1.02</td>
<td>1286 (39)</td>
<td>0.83</td>
</tr>
</tbody>
</table>

### Panel Fixed Effects Results

\[ \Delta p_{j,t} = \alpha_i + \beta \Delta m_{j,t} + \delta \Delta y_{j,t} + \sum \gamma_t yeardum_t + \varepsilon_{j,t} \]

<table>
<thead>
<tr>
<th>Sample Restrictions</th>
<th>$\beta$</th>
<th>95% Confidence Interval</th>
<th>Obs (countries)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.82***</td>
<td>0.71 - 0.92</td>
<td>4766 (125)</td>
<td>0.71</td>
</tr>
<tr>
<td>for $\Delta m_{j,t} &lt; 0.20$</td>
<td>0.001</td>
<td>-0.07 - 0.08</td>
<td>3385 (125)</td>
<td>0.24</td>
</tr>
<tr>
<td>for $\Delta m_{j,t} &gt;= 0.20$</td>
<td>0.97***</td>
<td>0.88 - 1.05</td>
<td>1381 (111)</td>
<td>0.82</td>
</tr>
<tr>
<td>for $\Delta m_{ave,j} &lt; 0.20$</td>
<td>0.14**</td>
<td>0.09 - 0.19</td>
<td>3480 (86)</td>
<td>0.29</td>
</tr>
<tr>
<td>for $\Delta m_{ave,j} &gt;= 0.20$</td>
<td>0.89***</td>
<td>0.79 - 0.98</td>
<td>1286 (39)</td>
<td>0.82</td>
</tr>
</tbody>
</table>

**NOTE:** Standard errors clustered by country. Fixed Effects and year dummies included.

We see that we now reject $\beta = 1$ for the entire sample. For the high-money-growth subsamples, we do not reject $\beta = 1$ except in one case, that of the within-effects case using country average money growth to select the sample. However, the confidence interval gets very close to 1.0. For countries with low money growth, the value of $\beta$ is insignificant in two of the four cases, in the other two the confidence interval tops out below .30. The difference in weighting between the country-averaged cross-section and the annual panel sample could account for why we find support for the quantity theory with the long-run averaged data. However, with the decadal and half-decadal...
data, the high money growth observations get a smaller weighting than in the annual data – about 25% weight – yet the results in Row 1 where money growth is not restricted, we find larger $\beta$s for the decadal and half-decadal results.

To sum up, the quantity theory holds best under two conditions: a long time horizon and high money growth. This is not a surprise. The former is expected because over long time horizons shocks have a greater period over which to die out, and positive and negative shocks will cancel each other out. At high money growth, the signal is relatively strong compared to the noise, so firms and people are more likely to raise prices in conjunction with contemporaneous increases in the money supply.

5.2 Results with M1

The results using M1 as the monetary aggregate for the decade data, the half-decade data, and the annual data are quite similar to those with M2. Therefore, we summarize but do not report the results. Specifically:

1. For the entire sample of countries - 123 with $M_1$, the null hypothesis that $\beta = 1$ is not rejected at the 95% confidence level with the long-run data, the decade data or with the half-decade data. It is rejected at that confidence level for the annual data.

2. For the sample of high money growth countries, the null hypothesis that $\beta = 1$ is not rejected at the 95% confidence level with the long-run data,
the decade data, the half-decade data, or the annual data. Further, the estimates of $\beta$ appear invariant to the degree of time aggregation.

3. For the sample of low money growth countries, the null hypothesis that $\beta = 1$ is rejected at the 95% confidence level with the long-run data, the decade data, the half-decade data, or the annual data. Further, the estimates of $\beta$ decline as the degree of time aggregation is lessened.

6 Inflation Targeting

In the early 1990s, a number of countries announced that they would begin targeting inflation rates. The first, New Zealand, set a target range of 1 – 3 percent. The Swedish Central Bank states a goal of “around 2 percent”. This kind of vagueness is not unusual for central banks that profess to target inflation. Inflation targeting (or more generally, any monetary policy feedback rule that adjusts money growth in reaction to the rate of inflation) means that our equation 3 may suffer from simultaneous equations bias since there will also be causality from $\Delta p$ to $\Delta m$. This will mean that $\text{cov}(\Delta m \cdot \varepsilon) \neq 0$ violating the condition necessary for an unbiased estimate of $\beta$. We address the bias issue in Section 7, but here consider whether our data provides evidence that inflation-targeting has occurred in practice.

To investigate whether or not targeting matters for the money-inflation relationship, we created a dummy variable $itdum$ set to 1 for any country-year for which the country was classified as an inflation-targeter. Our sample
of 125 countries includes 24 countries that have been classified as “inflation targeters” by the IMF. First, to see if inflation targeting had a practical effect on policy, we ran the following OLS equation using annual data:

$$\Delta m_{j,t} = \theta_0 + \theta_1 itdum_{j,t} + \eta_{1j,t}$$

(4)

The parameter $\theta_0$ is the average money growth for country-years in which inflation targeting was absent, and $\theta_0 + \theta_1$ is the average for those country-years with inflation targeting. We are most concerned with the sign and significance of $\theta_1$: it should be negative and significantly different from zero. Table 6, Column (1), shows the result of this estimation. We put the p-value in brackets under the coefficient. We easily reject the hypothesis that $\theta_1 = 0$ and the effect is economically interesting as well: money growth is about half as high for those that target inflation. In Column (2), we do the same for the inflation rate. The results are similar: inflation fell substantially and significantly in those country-years for which inflation targeting was reported.

In Columns (3) and (4) we repeat this exercise for the subset of countries that, at some point in their history, were identified as inflation targeters. The results are similar but stronger: money growth and inflation were higher for this group when they were not targeting, and they fell to a greater extent after they began to target.

It is possible that something happened around 1990 that led countries to reduce money growth and inflation in the years that followed.\textsuperscript{14} Therefore, we

\textsuperscript{14}Rogoff (2003) argues that increased competition in goods and labor markets around
ask whether countries who are not inflation-targeters also exhibited a lower rate of money growth post-1989. To investigate this alternative, we construct a dummy $tdum$ that takes the value 1 for every year after 1989. Then, we ran the following regression for just those countries that never targeted inflation:

$$\Delta m_{j,t} = \theta_3 + \theta_4 t dum_{j,t} + \eta_{2j,t}$$ 

(5)

We expect that there is no change in money growth due to time alone, so that $\theta_4 = 0$. We see in Columns (5) and (6) of Table 6 that money growth and inflation did not change appreciably for this group after 1990. These results taken together suggest that central banks that began a policy of inflation-targeting did in fact reduce their money growth rates and that the policy may be responsible for attenuating the one-to-one relationship between inflation and money growth for the low money growth countries.

7 Analysis of Bias for Low Money Growth Countries

Our results in Sections 4 and 5 reveal a pattern in the estimates of $\beta$ for the low money growth countries: $\beta$ declines as the degree of time aggregation.

---

15 We repeated this exercise using M1 money growth. The results are essentially the same, with one exception: money growth actually increased after 1989 for those countries that never targeted inflation.
Table 6: Inflation Targeting with M2

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Ever-Targeter</td>
<td>Never-Targeter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent Variable</td>
<td>∆m</td>
<td>∆p</td>
<td>∆m</td>
<td>∆p</td>
<td>∆m</td>
<td>∆p</td>
</tr>
<tr>
<td>Constant</td>
<td>0.186***</td>
<td>0.128***</td>
<td>0.249</td>
<td>0.188***</td>
<td>0.168***</td>
<td>0.115***</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>itdum θ₁</td>
<td>-0.085***</td>
<td>-0.091***</td>
<td>-0.148***</td>
<td>-0.150***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t dum θ₄</td>
<td></td>
<td>0.007</td>
<td>0.0009</td>
<td></td>
<td>[0.378]</td>
<td>[0.908]</td>
</tr>
<tr>
<td>N</td>
<td>4766</td>
<td>4766</td>
<td>1110</td>
<td>1110</td>
<td>3656</td>
<td>3656</td>
</tr>
</tbody>
</table>

Inflation declines. With the cross-section data, the estimate of \( \beta \) is 0.82 with a confidence interval very close to 1; with the annual data, \( \beta \) has declined to 0.25 or less. We suspect two factors are at work: simultaneous equations bias for inflation targeters (whether announced or not) and autocorrelation in money growth. We show below that under certain conditions, these two factors together can contribute to a downward bias in \( \beta \) that is more severe with data that has a smaller degree of time averaging. Our derivation of the bias is worked out for a single-country case, but we believe it extends to a panel. (Appendix 8 provides a full derivation).

In Section 6 we noted that inflation-targeting may introduce a non-zero correlation between contemporaneous outcomes of the regressor \( \Delta m_{jt} \) in equation 3 and \( \varepsilon_{jt} \). If inflation targeters adjust money growth over the year
in reaction to inflation over the year, the strongest bias will be present in the annual data. Further, inflation targeters are likely to fall in the low money growth group of countries where we observe the bias in $\beta$ to be the strongest. This may happen because for inflation-targeters, $cov(\Delta m \cdot \varepsilon) < 0$.

8 Conclusion

The quantity theory of money, in its simplest form, makes the prediction that changes in the rate of inflation should equal the change in the rate of money growth – provided that velocity is by nature uncorrelated both of these. What is not clear in the theory is the time horizon over which it is assumed to hold. Few would argue, for instance, that the daily inflation rate conforms to the quantity theory, but horizons of a decade or more are usually assumed to be ample to either confirm or reject the theory.

In this paper, we tested the quantity theory over four horizons: the cross section average, a panel of calendar decades, a panel of calendar half-decades, and annually. In each case, the regressions were run without restrictions on the coefficients so that we could test whether or not the coefficients on money and output growth differed from unity. We found a general pattern, but with some exceptions: in the whole sample, and in countries where money growth was high (averaging over 20 percent per year over the sample data), the coefficient on money growth did not differ significantly from unity. Where money growth was low, however, the confidence interval for $\beta$ was bounded
between zero and one. This was true for both pooled OLS and panel methods.

Inflation targeting may account for the failure of the quantity theory at low money growth rates. If monetary policy is actively used to offset price changes stemming from shocks to money demand, we would expect to see a smaller and less precise relationship between money growth – which could be highly variable – and price growth – which ideally would be zero. Using data from the IMF that identifies countries that target inflation – and the dates that it began – we were able to show that the coefficient on money growth was substantially lower for countries who say they actively stabilize the rate of inflation.

Our work shows two things. First, that countries with high inflation can bring it down by reducing the rate of money creation. This may be difficult, of course, and does not take into account the ways that money growth may be endogenous. Second, we have shown that inflation targeting may be a viable policy, in the sense that inflation in these countries is much less variable than money growth.

Appendix: Bias with Annual and Averaged Data

A Estimates of $\beta$ using Annual Data

Generically, our regression is:

$$y_t = \beta x_t + v_t$$

(6)
where $y$ is the inflation rate, $x$ is the growth rate of money and $v$ is the error term. We suppress the constant and other covariates to simplify the exposition.

The formula for $\beta$ using annual data is given by:

$$\beta_{ann} = \frac{Cov(xy)}{Var(x)} \quad (7)$$

$$\beta_{ann} = \frac{E(xy) - E(x)E(y)}{[E(xx) - E(x)E(x)]} \quad (8)$$

$$\beta_{ann} = \frac{E(xy) - \mu_x \mu_y}{[E(x^2) - \mu_x \mu_x]} \quad (9)$$

Now, let’s substitute equation 6 noting that $\mu_y = \beta \mu_x$. We then have:

$$\beta_{ann} = \frac{E(x(\beta x + v) - \beta \mu_x \mu_x)}{[E(x^2) - \mu_x \mu_x]} \quad (10)$$

$$\beta_{ann} = \frac{\beta(E(xx) - \mu_x \mu_x) + E(xv)}{[E(x^2) - \mu_x \mu_x]} \quad (11)$$

$$\beta_{ann} = \frac{\beta(E(x^2) - \mu_x \mu_x) + E(xv)}{[E(x^2) - \mu_x \mu_x]} \quad (12)$$

$$\beta_{ann} = \beta + \frac{E(xv)}{[E(x^2) - \mu_x \mu_x]} \quad (13)$$
For example, let $x = [x_1, x_2, x_3, x_4, x_5, x_6]$ or $x_t$ where $t = 1, ..., 6$ annual observations. (This will be important in Section B below). Then, we can write:

Note that $E(xy)$ is the mean of $(xy)$ and $E(x^2)$ is the mean of $x^2$. That is:

$$E(xy) = \left(\sum_{t=1}^{6} (x_ty_t)/6\right) = (x_1y_1 + ... + x_6y_6)/6 \quad (14)$$

$$E(x^2) = \left(\sum_{t=1}^{6} (x_t^2)/6\right) = (x_1^2 + ... + x_6^2)/6 \quad (15)$$

Thus, the estimate of $\beta$ using annual data making our substitutions for $E(xy)$ and $E(x^2)$ can be written as:

$$\beta_{ann} = \beta + \left[\sum_{t=1}^{6} x_tv_t/6\right]/[\left(\sum_{t=1}^{6} x_t^2/6 - \mu_x\mu_x\right] \quad (16)$$

Any bias in $\beta$ therefore occurs if the term in brackets is non-zero. The null hypothesis, using the Quantity Theory of Money is that the true value of $\beta = 1$. Thus, our estimate, $\beta_{ann}$ will be biased below 1 if the term in brackets is negative. It will be biased above 1 if the term in brackets is positive. We isolate the bias as:

$$bias_{ann} = \left[\sum_{t=1}^{6} x_tv_t/6\right]/[\left(\sum_{t=1}^{6} x_t^2/6 - \mu_x\mu_x\right] \quad (17)$$
This can be conventionally expressed as:

\[ \text{bias}_{ann} = \frac{\text{cov}(xv)}{\text{var}(x)} \quad (18) \]

### A.1 Case 1: No Contemporaneous Correlation between \(x\) and \(v\)

If we assume NO correlation between \(x\) and \(v\) which means \(\text{cov}(xv) \equiv \sum_{t=1}^{6} x_t v_t = 0\), we get:

\[ \beta_{ann} = \beta \quad (19) \]

\[ \text{bias}_{ann} = 0 \quad (20) \]

### A.2 Case 2: Contemporaneous Correlation between \(x\) and \(v\)

Contemporaneous correlation between \(x\) and \(v\) may occur if there is a monetary policy feedback rule for inflation. This will mean that \(\text{cov}(xv) \equiv \sum_{t=1}^{6} x_t v_t \neq 0\).

\[ \beta_{ann} = \beta + \text{bias}_{ann} \quad (21) \]
and:

\begin{align*}
\text{bias}_{\text{ann}} > 0 \text{ if } \text{cov}(xv) > 0 \\
\text{bias}_{\text{ann}} < 0 \text{ if } \text{cov}(xv) < 0
\end{align*}

(22)

(23)

Since the monetary policy feedback rule will generally assume that the money supply declines when the inflation rate rises and vice-versa, we expect that \( \text{cov}(xv) < 0 \) which means that \( \beta_{\text{ann}} < \beta = 1 \). More importantly for explaining our results as we use data sets with greater degrees of time-aggregation, monetary policy feedback rules that target inflation will on average reduce their money growth. This means that countries in the low money growth category are more likely to be inflation-targeters. Thus, \( \text{cov}(xv) < 0 \) is more likely at low money growth rates than at high rates. We could also argue that countries with very high rates of money growth have essentially adopted a policy of letting the money supply outstrip the inflation rate. If true, then \( \text{cov}(xv) > 0 \) at high rates of inflation so it is possible that \( \beta_{\text{ann}} > \beta = 1 \).

**B  Estimates of \( \beta \) using Averaged Data**

Now, let's compare \( \beta_{\text{ann}} \) in equation 12 to an estimate using the same annual data averaged over intervals, \( \beta_{\text{ave}} \). Our regression specification would be:
\[ Y_k = \beta_{\text{ave}} X_k + V_k \] (24)

Let’s use our \( T = 6 \) annual observations and average them over \( J = 2 \) years each. This establishes the number of intervals over which averaging is done, i.e. \( T/J = K \). In our case \( K = 3 \). That is, we have 3 observatoins each on \( X \) and \( Y \). Now, let’s use the counter \( j = 1, 2 \) years and \( k = 1, 2, 3 \) intervals.

Let \( X = [X_1 X_2 X_3] \) or \( X_k \), \( Y = [Y_1 Y_2 Y_3] \) or \( Y_k \) and \( V = [V_1 V_2 V_3] \) or \( V_k \). That is, each element of \( X, Y, \) and \( V \) is the two-year average of the annual observations such that:

\[
X_1 = (x_1 + x_2)/2 \quad \text{and} \quad X_2 = (x_3 + x_4)/2 \quad \text{and} \quad X_3 = (x_5 + x_6)/2 \\
Y_1 = (y_1 + y_2)/2 \quad \text{and} \quad Y_2 = (y_3 + y_4)/2 \quad \text{and} \quad Y_3 = (y_5 + y_6)/2 \\
V_1 = (v_1 + v_2)/2 \quad \text{and} \quad V_2 = (v_3 + v_4)/2 \quad \text{and} \quad V_3 = (v_5 + v_6)/2
\]

The formula for \( \beta \) using data averaged from the annual data is thus given by:

\[
\beta_{\text{ave}} = \frac{\text{Cov}(XY)}{\text{Var}(X)}
\] (25)

which can be re-written as:

\[
\beta_{\text{ave}} = \frac{[E(XY) - E(X)E(Y)]/[E(XX) - E(X)E(X)]}{[E(XX) - E(X)E(X)]}
\] (26)
which can be expressed as:

\[ \beta_{ave} = \frac{E(XY) - \mu_X \mu_Y}{E(X^2) - \mu_X \mu_X} \]  (27)

Now, let’s use equation 24 to substitute in for \( Y \) and note that \( \mu_Y = \beta \mu_X \).

We then have:

\[ \beta_{ave} = \frac{E(X(\beta X + V) - \beta \mu_X \mu_X)}{E(X^2) - \mu_X \mu_X} \]  (28)

\[ \beta_{ave} = [\beta(E(XX) - \mu_X \mu_X)] + E(XV)/[E(X^2) - \mu_X \mu_X] \]  (29)

\[ \beta_{ave} = [\beta(E(X^2) - \mu_X \mu_X)] + E(XV)/[E(X^2) - \mu_X \mu_X] \]  (30)

\[ \beta_{ave} = \beta + E(XV)/[E(X^2) - \mu_X \mu_X] \]  (31)

This is the familiar formula for \( \beta \). We see that the second term, the bias portion, depends on whether \( E(XV) \) is equal to zero or not. The magnitude of the bias depends on both the numerator and denominator. Our assumption about contemporaneous correlation discussed in Section A means that
the correlation occurs at the annual level. Therefore, we will need to look explicitly at the construction of \( X_j \) and \( V_j \) from the annual data. Because we are working with observations that have been averaged over two years for \( K = 3 \) intervals, the expected value of \( E(XV) \) and \( E(X^2) \) are more complicated expressions than when we used the annual data. Let’s unpack \( X \) and \( V \) in equation 31 to their annual components and see what we get for the bias portion for \( \beta_{\text{ave}} \):

\[
\text{bias}_{\text{ave}} = \frac{\sum_{j=1}^{3} (X_j V_j) / 3}{\sum_{j=1}^{3} X_j^2 / 3 - \mu_X \mu_X} \quad (32)
\]

Let’s look at the numerator in equation 32 and make use of \( X_1 = (x_1 + x_2) / 2 \), etc. The numerator of equation 32 can be written as:

\[
\sum_{j=1}^{3} (X_j V_j) / 3 = 1/3\left\{ (x_1 + x_2)(v_1 + v_2)/2^2 + (x_3 + x_4)(v_3 + v_4)/2^2 + (x_5 + x_6)(v_5 + v_6)/2^2 \right\} / \quad (33)
\]

which can be expressed as:

\[
1/3 \sum_{t=1}^{6} x_tv_t/4 + (x_1 v_2 + x_2 v_1 + x_3 v_4 + x_4 v_3 + x_5 v_6 + x_6 v_5)/4 \quad (34)
\]

The denominator of equation 32 can be written as:
\[ \frac{1}{3} \left\{ (x_1 + x_2)(x_1 + x_2)/4 + (x_3 + x_4)(x_3 + x_4)/4 + (x_5 + x_6)(x_5 + x_6)/4 \right\} - \mu_X \mu_X \] (35)

which can be further re-arranged as:

\[ \left\{ \sum_{t=1}^{6} \left( \frac{x_t^2}{12} \right) + \frac{2(x_1x_2 + x_3x_4 + x_5x_6)}{12} \right\} - \mu_X \mu_X \} \] (36)

The bias in \( \beta_{ave} \) in equation 32 therefore comes from the ratio of numerator and denominator:

\[
\text{bias}_{ave} = \frac{\{(1/3) \sum_{t=1}^{6} x_t v_t/4 \} + \frac{1}{3}(x_1 v_2 + x_2 v_1 + x_3 v_4 + x_4 v_3 + x_5 v_6 + x_6 v_5)/4}{\left\{ \sum_{t=1}^{6} \left( \frac{x_t^2}{4} \right) + \frac{1}{3}(2x_1x_2 + 2x_3x_4 + 2x_5x_6)/4 \right\} - \mu_X \mu_X \} \] (37)

If we run the 1/3 through the necessary terms, we get:

\[
\text{bias}_{ave} = \frac{\{(\sum_{t=1}^{6} x_t v_t/12) + (x_1 v_2 + x_2 v_1 + x_3 v_4 + x_4 v_3 + x_5 v_6 + x_6 v_5)/12 \} \} / \left\{ \left( \sum_{t=1}^{6} \left( \frac{x_t^2}{12} \right) + \frac{2(x_1x_2 + x_3x_4 + x_5x_6)}{12} \right) - \mu_X \mu_X \right\} \] (38)
We note that $\mu_X\mu_X = \mu_x\mu_x$. Notice that there is some similarity of equation 37 or equation 38 to the equation 21. They differ by crossproducts of $x_tv_{\neq t}$, by the last term in the denominator, and by the divisors. The divisor here differs by a factor of 2. That is, with annual data, any squares are summed over 6 observations whereas with the averaged data the sums are over 3 observations. Further, with annual data, no crossproduct terms are produced. For example, since $X_1 = (x_1 + x_2)/2$, $X_1^2 = (x_1^2 + x_2^2 + 2x_1x_2)/4$. With annual data, when the equivalent first observation in the sample is squared, we get $x_1^2$. And, the observations are averaged over 2 years but are squared in the calculation so that $4 = 2^2$. Thus, we get a divisor, where it appears, of 12 ($2^23$), instead of 6.

$$bias_{\text{ave}} = \frac{\text{cov}(XV)}{\text{var}(X)}$$

(39)

Now, the numerator of equation 37 is $\text{cov}(XV)$ which is a covariance that includes crossproducts. If we assert that $\text{cov}(x_tv_{\neq t}) = 0$, then $\text{cov}(XV) = \text{cov}(xv)$. If we further assume that $\text{cov}(x_tv_{\neq t}) = 0$, then $\text{var}(X) = \text{var}(x)$. Under these assumptions, we have a special case where:

$$bias_{\text{ave}} = bias_{\text{ann}}$$

(40)
B.1 What happens to the bias in $\beta_{ave}$ if $x$ and $v$ are NOT correlated?

Equation 38 shows that if $x$ and $v$ are not correlated for any $t$, then there is no bias. This is true regardless of whether $x_t$ is correlated with $x_{t-1}$.

B.2 What happens to the bias in $\beta_{ave}$ if $x$ and $v$ are contemporaneously correlated?

If $x$ and $v$ are correlated, $E(x_tv_t) \neq 0$, there are several possible outcomes depending on what is assumed about the correlation of $x_t$ with $x_{t-1}$. Recall that with inflation-targeting, we presume that $x$ and $v$ are negatively correlated for contemporaneous outcomes of $x$ and $v$. This means the bias in $\beta_{ave}$ will be downward as with $\beta_{ann}$. However, the formula for $bias_{ave}$ is much more complicated because it includes crossproducts of $xv$ and crossproducts of $xx$.

We use the term crossproducts to mean that the time $t$ subscripts on $x$ and $v$ are not the same, i.e. they are not contemporaneous. We note here that the crossproducts in the $bias_{ave}$ formula are not straight crossproducts because not all combinations of non-contemporaneous pairs are included. (See equation 38).

Our empirical results show that when money growth is less than 20%, our estimate of $\beta$ using annual data, $\beta_{ann}$, is smaller than any of our estimates of $\beta$ using our averaged datasets. That is, the bias declines (which means $\beta \to 1$) as the degree of time aggregation rises. We observe that as the extent
of time-averaging increases from annual to half-decade to decade to long run (full sample average), the estimates of $\beta$ with pooled data increase from 0.25 to 0.51 to 0.64 to 0.82. This may also mean that the bias in our estimate of $\beta$ declines as the degree of time averaging rises.

The difference in the magnitude of $bias_{ave}$ and $bias_{ann}$ assuming that $x$ and $v$ are contemporaneously correlated will depend on whether $x_t$ is autocorrelated. First, we can see from equation 38 that the denominator includes an additional term when there is autocorrelation in $x$. This is not present when there is no autocorrelation. Second, if there is autocorrelation in $x_t$, and contemporaneous correlation between $x_t$ and $v_t$, there will be correlation between $x_t$ and $v_{\neq t}$. This will create an additional term in the numerator.

B.2.1 $x_t$ and $v_t$ are contemporaneously correlated but $x_t$ and $x_{t-1}$ are NOT correlated

With contemporaneous correlation only between $x$ and $v$, equation 38 shows the formula reduces to:

$$bias_{ave} = \left(\sum_{t=1}^{6} x_t v_t / 12\right) / \left\{\sum_{t=1}^{6} (x_t^2 / 12) + 2(x_1 x_2 + x_3 x_4 + x_5 x_6) / 12 - \mu_X \mu_X\right\}$$

(41)

The difference between $\beta_{ann}$ in equation 21 and $\beta_{ave}$ is the additional term $2(x_1 x_2 + x_3 x_4 + x_5 x_6) / 12$ in the divisor and the number 12 versus 6. Depending on the influence of these factors, $bias_{ave}$ could be greater or smaller.
than $bias_{ann}$ and depending on assumptions we make about the crossproducts of $xv$ and $xx$ and whether they sum to a positive or negative number, the bias could go any direction and its magnitude could be large or small.

**B.2.2 $x_t$ and $v_t$ are correlated but $x_t$ and $x_{t-1}$ are correlated**

I believe that if $x_t$ and $v_t$ are correlated and $x_t$ and $x_{t-1}$ are correlated, then $x_tv\neq t$ will also be correlated. This means that all of the terms in equation 38 will be present and the direction of the bias will depend on their relative size.

**B.3 General Formula for the bias proportion with averaged data**

Using equations 37-38 as a guide, I will use to pattern a general formula for the bias. I will use the following notation:

$T$ is the total number of annual (smallest period) observations with $t = 1, ..., T$

$J$ is the number of periods averaged over. $J \geq 2$.

$K$ is the number of intervals of averaged observations. $K = T/J$ with $k = 1, ..., K$.

\[
bias_{ave} = \frac{(1/K \sum_{t=1}^{T} x_tv_t/J^2) + 1/K \sum_{t=1,odd}^{T-1} x_tv_{t+1} + \sum_{t=2,even}^{T} x_tv_{t-1}/J^2}{...}
\]
\[
\{[1/K \sum_{t=1}^{T} (x_t^2/J^2) + 2/K( \sum_{t=1, odd}^{T-1} x_t x_{t+1})/J^2] - \mu X \mu X \}
\] (42)

This can be re-arranged as:

\[
\text{bias}_{\text{ave}} = \{(1/KJ^2 \sum_{t=1}^{T} x_t v_t) + 1/KJ^2( \sum_{t=1, odd}^{T-1} x_t v_{t+1} + \sum_{t=2, even}^{T} x_t v_{t-1})\}/
\]

\[
\{[1/KJ^2 \sum_{t=1}^{T} x_t^2] + 2( \sum_{t=1, odd}^{T-1} x_t x_{t+1})] - \mu X \mu X \}
\] (43)
References


