State High School Mathematics Tournament

University of South Carolina

February 3, 2018

Tiebreaker Rules

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- Try to solve it **approximately**, as accurately as you can, and make an educated guess.
- The answer(s) **closest to the truth** (in either direction) win the tiebreaker.

Tiebreaker Question

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- We have
$$m^2 + n^2 \leq 2018.$$

Tiebreaker Answer

Answer. 3176 (ask a computer).

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- Pairs (m, n) with $m^2 + n^2 \leq 2018$ correspond to **lattice points** inside (or on) the circle $m^2 + n^2 \leq 2018$.

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- The number of such lattice points is approximately the area of the circle, 2018π.

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- $m^2 + n^2$ will be one plus an integer multiple of 4 if and only if m and n are of opposite signs, so we should count only half the lattice points.

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- Doing better requires brute force or a computer.