Quadratic-Nonlinearity Power-Index Spectrum and Its Application in Condition Based Maintenance (CBM) of Helicopter Drive Trains

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Abstract—This paper introduces a quadratic-nonlinearity powers-index spectrum (QNLPI(f)) measure that describes quantitatively how much of the mean square power at certain frequency $f$ is generated by nonlinear quadratic interaction between different frequencies inside signal spectrum. The proposed index QNLPI(f) is based on the bicoherence spectrum, and the index can be simply seen as summary of the information contained in the bicoherence spectrum in two dimensional graph which makes it easier to interpret. The proposed index is studied first using computer generated data and then applied to real-world vibration data from a helicopter drive train to characterize different mechanical faults. This work advances the development of health indicators based on higher order statistics to assess fault conditions in mechanical systems.

I. INTRODUCTION

Spectral analysis of vibration signals is the most common and popular technique used in the field of condition monitoring of rotating machinery [1], [2]. Unfortunately, conventional spectral analysis techniques are of limited value when various spectral components interact with one another due to some nonlinear or parametric processes. In such a case, higher order spectral techniques (HOS) are necessary to accurately and completely characterize the fluctuating data in frequency domain [3]. Since nonlinearities result in new spectral component being formed which are phase coupled with the interacted components, bispectrum (third-order spectrum) and its normalized version, bicoherence, are used to detect such quadratic phase coupling (QPC) [4]. The QPC is an indicator of nonlinear signal production mechanism which is very common when machinery faults occur [5], [6].

However, using HOS becomes a challenge in practical applications when the experimental data has a wide range of frequency interactions such as transmission gear box and hanger bearing spectra of mechanical system. The three dimensional nature of the bicoherence requires careful design of the view and quite expert personnel to interpret the results in the frequency domain. Motivated by these observations, we propose and investigate a two dimensional quadratic-nonlinearity power-index spectrum (QNLPI(f)) based on the bicoherence function. The proposed QNLPI(f) spectrum quantifies the fraction of power (mean square) at a certain frequency $f$ due to all the possible combinations of nonlinear quadratic couplings that may cause the creation of the frequency $f$. The proposed QNLPI(f) summarizes the information in the 3D bicoherence spectrum into 2D graph, which makes it easy to use and interpret. This is carried out by plotting the accumulative resultant power spectrum of the nonlinear interaction instead of the interacting frequencies in the bicoherence case, as will be discussed in section II.

In this paper, the proposed concept of the QNLPI(f) is applied to vibration data collected from helicopter drive-train at the CBM center at USC. As will be shown, using QNLPI(f) spectrum helps to gain more details about nonlinear harmonic generation patterns that we use to distinguish between different cases of mechanical faults such as (1) misalignment, (2) unbalance, and (3) combination of misalignment and unbalance of shafts.

II. CONCEPT OF QUADRATIC-NONLINEARITY POWER-INDEX SPECTRUM

The bispectrum $B(f_1, f_2)$ for a zero mean stationary random signal $x(t)$ is defined as follows [3].

$$B(f_1, f_2) = E\{X(f_1)X(f_2)X^*(f_3)\} : f_3 = f_1 + f_2 \tag{1}$$

where $E\{\}$ denotes a statistical expected value operator. $X(f)$ is the discrete Fourier transform (DFT) of $x(t)$, and * denotes a complex conjugate.

The definition of the bispectrum in (1) shows that $B(f_1, f_2)$ will be zero unless the following two conditions are met:

1. Waves must be present at the frequencies $f_1$, $f_2$, and $f_1 + f_2$. That is, $X(f_1)$, $X(f_2)$, and $X(f_1 + f_2)$ must be non-zero, and
2. A phase coherence must be present between the three frequencies $f_1$, $f_2$, and $f_1 + f_2$.

Symmetry properties of the bispectrum in addition to Nyquist frequency limit imply that it is sufficient to compute the bispectrum over the triangle area $A$ in the $f_1 - f_2$ plane, shown in Figure 1 where $f_N$ is the Nyquist frequency.

The magnitude of the bispectrum at coordinate point $(f_1, f_2)$ measures the degree of phase coherence between the three frequency components $f_1$, $f_2$, and $f_3$. However, this magnitude is also dependent on the magnitude of the relevant
Fourier coefficients. Therefore, a common function used to normalize the bispectrum is the bicoherence \( b(f_1, f_2) \) [3], [6].

\[
b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{E\{|X(f_1)X(f_2)|^2\} E\{|X(f_3)|^2\}} : f_3 = f_1 + f_2
\]  

(2)

The bicoherence is independent of the magnitude of the Fourier transform and bounded by \( 0 \leq b(f_1, f_2) \leq 1 \). Moreover, it has been proven in [3] that the squared bicoherence \( b^2(f_1, f_2) \) quantifies the fraction of power (mean square) at \( f_3 = f_1 + f_2 \) (result) due to the coupling of the waves at \( f_1 \) and \( f_2 \) (source).

The Quadratic-Nonlinearity Power-Index spectrum \( QNLPI(f) \) is a conversion of the 3D bicoherence spectrum (showing the source of interaction) into 2D spectrum (showing the accumulative results). This can be done by summing the bicoherence values at all frequency combinations that result in \( f \), as follows:

\[
\sum_{f_1+f_2=f} b^2(f_1, f_2)
\]  

(3)

In other words, adding the values of \( b^2(f_1, f_2) \) along a straight line in \( f_1 - f_2 \) plane with a slope of \(-45^\circ\) represented mathematically by

\[
f_2 = f - f_1
\]  

(4)

where, \( f \) is variable determines the point of interest in the horizontal axis of the \( QNLPI(f) \) spectrum. However, we should be very careful when we apply the summation in (3) to the region of computation indicated by the triangle A shown in Figure 1. In fact, the bicoherence of interacted frequencies in the fourth quadrant (positive \( f_1 \) and negative \( f_2 \)) has a redundant copy in this A region due to symmetry properties of the bicoherence. Therefore, the region of computation in \( f_1 - f_2 \) plane is modified to fully map the quadratic interaction between different frequencies as shown in Figure 1. Area covered by triangle B maps the difference part of the interaction between two frequencies, while area covered by triangle A maps only the sum part.

Based on this new region of computation, summation (3) is described by the following equation:

\[
QNLPI(f) = \sum_{n=0}^{N-1} b^2((\frac{f}{2} + n\Delta f) - (\frac{f}{2} - n\Delta f))
\]  

(5)

where \( N = f_N/\Delta f \), and \( \Delta f \) is the elementary band width determined from the accuracy of DFT calculation. The frequency resolution \( \Delta f \) should be smaller than the difference between the smallest two frequencies expected to interact in any particular case. Thus, it is very important that we carefully design the sampling rate and the record length \( T \) of the signal under test since those two parameter has direct potential effect on \( \Delta f \).

Different computer-generated test signals have been used to study the proposed \( QNLPI(f) \). Simple case is shown in equation (6).

\[
x(t) = A_b \cos(2\pi f_b t + \theta_b) + A_c \cos(2\pi f_c t + \theta_c) + A_{bc} \cos(2\pi f_b t + \theta_b) \times \cos(2\pi f_c t + \theta_c) + A_d \cos(2\pi f_d t + \theta_d) + n(t)
\]  

(6)

\( A_b = A_c = A_d = 2 \), \( A_{bc} = 4 \), sampling frequency \( f_S = 2f_N = 4.8kHz \), \( f_b/f_N = 0.22 \), \( f_c/f_N = 0.375 \), and \( f_d = f_b + f_c \). All phases are independently taken from a set of uniformly distributed random numbers, and, \( n(t) \) is a small amplitude Gaussian noise (-20dB).

In this case, half of the power at \( f_d \) is due to the quadratic nonlinear interaction between \( f_b \) and \( f_c \). The power at \( f_a = f_c - f_b \) is totally generated by the interaction. The power spectrum of the test signal, the modified bicoherence \( b^2(f_1, f_2) \),
Fig. 4. QNLPI spectrum for test signal in equation (6)

and the quadratic-nonlinearity power-index $QNLPI(f)$ are shown in Figures 2, 3 and 4 respectively.

Another test signal has been created to illustrate the usefulness of the proposed $QNLPI(f)$ as shown in (7).

$$x(t) = A_b \cos(2\pi f_b t + \theta_b) + A_c \cos(2\pi f_c t + \theta_c) + A_g \cos(2\pi f_g t + \theta_g) + A_{bc} \cos(2\pi f_{bc} t + \theta_b) \times \cos(2\pi f_{bc} t + \theta_c) + A_{cg} \cos(2\pi f_{cg} t + \theta_c) \times \cos(2\pi f_{cg} t + \theta_g) + n(t) \tag{7}$$

$A_b = A_c = A_d = A_g = 2$, $A_{bc} = A_{cg} = 4$, sampling frequency $f_s = 2f_N = 4.8KHz$, $f_b / f_N = 0.22$, $f_c / f_N = 0.375$, $f_g / f_N = 0.292$, $f_{bc} / f_N = 0.303$ and $f_d = f_b + f_c + f_g$. All phases are independently taken from a set of uniformly distributed random numbers. And, $n(t)$ is a small amplitude Gaussian noise (-20dB).

III. EXPERIMENTAL SET UP AND DATA DESCRIPTION

The CBM center at The University of South Carolina has an AH-64 helicopter tail rotor driveshaft apparatus for on-site data collection and analysis. As shown in Figure 8, the apparatus is a dynamometric configuration which includes an AC induction motor rated at 400hp controlled by variable frequency drive to provide input drive to the configuration, a multi-shaft drive train supported by hanger bearings, flex couplings at shaft joining points, two gearboxes, and an absorption motor of matching rating to simulate the torque loads that would be applied by the tail rotor blade.
Signals denoted as FHB and AHB, measured at forward and aft hanger bearings vibrations respectively, have been previously gathered at two minutes intervals at a sampling rate of 48 kHz over the course of thirty minute baseline test runs [7]. Measurements are taken for each drive-shafts setting under test which include baseline shaft configuration, unbalance in different shafts configuration, and shaft misalignment, all common issues on AH-64 drivetrains. Misalignment between drive-shafts is tested at 1.3° between centerlines of drive shafts #3 and #4, and 1.3° between drive shafts #4 and #5. Unbalance is also introduced to drive shafts #3, #4 and #5 by 0.140 oz-in, 0.135 oz-in 0.190 oz-in respectively. Combination of misalignment and unbalance are considered as summarized in Table I. Each setting has been given code to distinguish different tested bearings and faulted cases.

<table>
<thead>
<tr>
<th>Shaft Status</th>
<th>Balanced</th>
<th>Unbalanced(4&amp;5)</th>
<th>Unbalanced(3,4&amp;5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aligned</td>
<td>00321</td>
<td>10321</td>
<td>N/A</td>
</tr>
<tr>
<td>Misaligned</td>
<td>20321</td>
<td>30321</td>
<td>40321</td>
</tr>
</tbody>
</table>

IV. RESULTS OF ANALYZING DRIVE-SHAFT FAULTS USING QNLPI

Due to the loading scheme of the TRDT test stand with the intermediate gear box (IGB) and the output motor, the 3rd harmonic of the tail rotor drive shaft (243 Hz) is dominating the power spectrum of the AHB in all tested cases with some other different harmonics in each setting, as shown in Figures 9 and 10. The power spectra of the baseline (00321) and the misaligned (20321) cases in Figure 9 have the same dominating spectral peaks with very slight changes in the minor peaks. A similar situation occurs when we compare the unbalanced (10321) and the misaligned-unbalanced (30321) cases in Figure 10. It is not an easy task to distinguish between different cases using only the traditional power spectrum.

The bicoherence investigates the nonlinear relationship between different frequencies inside signal spectrum regardless of their magnitudes. The squared bicoherence spectrum of the baseline case is shown in Figure 11 with the quadratic nonlinear interaction between the 3rd and the 4th harmonics is dominating in this case. The result of this nonlinear interaction is the new generated 1st and 7th harmonics as shown in Figure 12(a). It is worth to note that the value of the QNLPI at the new generated frequencies, 1st and 7th harmonics, are 0.68 and 0.77 respectively. This means that most of the power at these frequencies is a direct result of the quadratic interaction. The remaining fraction of the power may be independently excited or coming from other forms of nonlinearities.

Due to different physical setting, A different interaction pattern is exist in the case of misalignment as shown in Figure 13. In this case, quadratic nonlinear interaction between the 3rd and the 1st harmonics is dominating. As a result of this interaction, 2nd and 4th harmonics are generated with
power fraction of 0.72 and 0.64 respectively as shown in Figure 12(b). The results in Figures 11, 13 and 12 give us more details about the content of the power spectrum of the signal. Some frequencies in common between the baseline and misaligned cases have different origins. For example, the $1^{st}$ and the $4^{th}$ harmonics exchange there places as source/result of the interaction process with the $3^{rd}$ due to different physical setting of the rotating shaft.

The analysis of the vibration signal using the bicoherence spectrum becomes more challenging when a combination of misalignment and unbalance is introduced the rotating shaft as shown in Figure 14. It is obvious that the nonlinearity of the system increases causing different combinations of frequency-frequency interaction. In such cases, it is much easier to deal with the proposed QNLPI spectrum. As mentioned before, the QNLPI provide a summary bicoherence spectrum by showing the accumulative results of the nonlinear interaction in single frequency spectrum.

Comparing the QNLPI of the unbalance case shown in Figure 15(a) with the baseline in Figure 12(a), we can see a slightly more interaction introduced in the case of the unbalance. The $4^{th}$ harmonic interacts with both $3^{rd}$ and $9^{th}$ producing a series of odd harmonics at $1^{st}$, $7^{th}$, $5^{th}$, and $13^{th}$. The increasing production of odd harmonics through the nonlinear interaction is likely a sign of unbalance. On the other hand, as discussed above, the production of even harmonics is likely a sign of misalignment. Therefore, A combination of unbalance and misalignment will produce a variety of odd/even harmonics of the drive shaft rotating frequency as can be seen in Figure 15(b) and (c).

V. CONCLUSION

Quadratic-nonlinearity powers-index ($QNLPI(f)$) measure has been proposed which provide a summary of the 3D bicoherence into 2D graph. Hence, fewer skills are needed from personnel dealing with the spectrum analysis. Using $QNLPI(f)$ spectrum helps to gain more details about nonlinear harmonic interaction/generation patterns which we used to distinguish between different cases of mechanical faults in a helicopter tail rotor drive train. The unique nonlinearity signature of each fault can be used to design more accurate diagnoses algorithms for the condition based maintenance (CMB) practice.

REFERENCES