Applications of Time-Frequency Analysis for Aging Aircraft Component Diagnostics and Prognostics

Kwangik Cho, David Coats, John Abrams, Nicholas Goodman †, Yong-June Shin, and Abdel E. Bayoumi †
Department of Electrical Engineering
† Department of Mechanical Engineering
The University of South Carolina
Columbia, SC 29208, USA

ABSTRACT

The classical time-frequency distributions represent time- and frequency-localized energy. However, it is not an easy task to analyze multiple signals that have been simultaneously collected. In this paper, a new concept of non-parametric detection and classification of the signals is proposed using the mutual information measures in the time-frequency domain. The time-frequency-based self and mutual information is defined in terms of cross time-frequency distribution. Based on the time-frequency mutual information theory, this paper presents applications of the proposed technique to real-world vibration data. The baseline and misaligned experimental settings are quantitatively distinguished by the proposed technique.

Keywords: Time-Frequency Information, Rényi Information, Cross Time-Frequency Distribution, Mutual Information, Condition Based Maintenance

1. INTRODUCTION

The standard maintenance practices in military aviation involve replacing existing parts after a certain time period or a certain number of operational hours. This practice is called time-based maintenance (TBM) and can lead to failures in critical parts due to unexpected wear, causing operational downtime and potential safety hazards. Therefore, instead of TBM, it is desirable to consider time- and use-based maintenance practices so that the critical parts are replaced before their full lifetimes for economic and safe operations. With this goal in mind, a new practice of condition-based maintenance (CBM) is proposed for military aviation fleet management. The goals of CBM involve changing the time- and reaction-based maintenance schedules into ones that are predictive and proactive. However, in order to achieve this innovative maintenance practice, data must be collected from vital operational components and analyzed to determine the current and future health of these components. In order to monitor the health status of the systems, a variety of signals are collected, including vibration, acoustic, and temperature. Thus, the first step in this process is to collect historical data on which a baseline of operation can be established. After the characteristics of the baseline are established, diagnostic algorithms can be developed to determine if problems exist on the specified component.

Previous studies on CBM have shown that the abnormality of the system is characterized by transient precursors in the signals. Through their use in detecting transient precursors, advanced signal processing techniques have contributed to develop diagnostics and prognostics algorithms for aging aircraft. In particular, time-frequency analysis is extremely useful to analyze the transient signature of the abnormality and its precursors. However, the applications of classical time-frequency analysis to CBM is limited in practice. The classical time-frequency distributions represent time- and frequency-localized energy. However, it is not an easy task to analyze multiple signals that are simultaneously collected in CBM systems. In this paper, we propose a new concept of non-parametric detection and classification of the signals. We define time-frequency based self and mutual information in order to classify the health status of the system components in Section 2. The experimental setup and data description are further described in Section 3. The effectiveness of the proposed technique is demonstrated in Section 4 through real-world vibration data. The conclusions and future work are presented in Section 5. Further author information: (Send correspondence to Yong-June Shin)
E-mail: shinjune@engr.sc.edu,
Telephone: +1 (803) 777-9569,
Address: 301 S. Main St. Columbia, SC 29208, U.S.A
provided in Section 3. The results and discussions are provided in Section 4, and conclusion of the paper is drawn in Section 5. Based on the time-frequency based mutual information theory, this paper presents applications of the proposed technique to the real-world vibration data.

2. TIME-FREQUENCY INFORMATION MEASURE

2.1. Rényi Information

Classical information measure of a continuous stochastic process is known as a Shannon information.\(^7\)

\[
H_x = -\int_{-\infty}^{\infty} f(x) \log_2 f(x) dx
\]

(1)

where the continuous function \(f(x)\) is a probability density function which is positive and bounded between 0 and 1. Williams, Brown, and Hero proposed a measure of time-frequency information by use of the generalized Rényi information.\(^8\) The definition of the generalized Rényi information of a continuous bivariate distribution \(P(x, y)\) is defined as follows:\(^9\)

\[
H_\alpha(P) := \frac{1}{1 - \alpha} \log_2 \frac{\iint P^\alpha(x, y) dxdy}{\iint P(x, y) dxdy}
\]

(2)

The definition of the generalized Rényi information can be extended by replacing the bivariate distribution \(P(x, y)\) with a Cohen’s class time-frequency distribution \(C_s(t, \omega)\) of signal \(s(t)\) with the following definition:\(^10\)

\[
C_s(t, \omega; \phi) = \frac{1}{4\pi^2} \iint s^*(u - \frac{\tau}{2}) s(u + \frac{\tau}{2}) e^{-j\theta t - j\omega \tau + j\theta u} d\theta d\tau du
\]

(3)

The order of the generalized Rényi information determined by parameter \(\alpha\) for the time-frequency distribution has been investigated by Baranuk et al.\(^8\) so that \(\alpha = 3\) is a reasonable selection of the order, with the exception of contrived counterexamples.\(^8\) Hence, the following information measure of time-frequency distribution will be utilized in this paper:

\[
H_\alpha(C_s) = \frac{1}{1 - \alpha} \log_2 \frac{\iint C_s^\alpha(t, \omega) dtd\omega}{\iint C_s(t, \omega) dtd\omega}
\]

(4)

The metric \(H_\alpha(C_s)\) defined in (4) measures the number of signal elements of \(s(t)\) over the time and frequency planes. The Rényi information measure is a meaningful measure of time-frequency distribution, but it is only defined for a single realization of signal, e.g. self-information. If we have a pair of signals closely related, how can we define or quantify the interactions in terms of information? We will investigate a generalization of the time-frequency information measure by introducing the mutual time-frequency information.

2.2. Mutual Time-Frequency Information Measure

In order to analyze the information of two closely spaced components, the classical mutual information of two random processes is extended to two time-frequency distribution functions. Let us consider the classical definition of the mutual information that might be extended to the measure of mutual information of the time-frequency distributions. The joint entropy\(^7\) \(H(X, Y)\) of a pair of continuous random variables \((X, Y)\) with a joint probability density function \(p(x, y)\) is defined as:

\[
H(X, Y) = -\iint p(x, y) \log_2 p(x, y) dxdy
\]

(5)

By chain rule,

\[
H(X, Y) = H(X) + H(Y|X)
\]

(6)

where \(H(Y|X)\) is the conditional entropy. Under the same conditions, the mutual information \(I(X; Y)\) is the relative
The relation of the mutual information \( I(X;Y) \) and joint entropy \( H(X,Y) \) is defined as follows:

\[
I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = H(X) + H(Y) - H(X,Y)
\]

(8)

\( J_{s_1s_2}(t,\omega;\phi) \) is the complex time-frequency distribution of the signal pairs \( S_1 \) and \( S_2 \), which is called cross time-frequency distribution. In this paper, \( J_{s_1s_2}(t,\omega;\phi) \) corresponds to the multiplication of the short time Fourier transform of \( S_1 \) and the complex conjugate of the short time Fourier transform of \( S_2 \). Consider a joint information of time-frequency distribution \( H_\alpha(J_{s_1s_2}) \) in terms of cross time-frequency distribution \( J_{s_1s_2}(t,\omega;\phi) \) as follows:

\[
H_\alpha(J_{s_1s_2}) = -\frac{1}{1-\alpha} \log_2 \frac{\int \int J_{s_1s_2}^\alpha(t,\omega)dtd\omega}{\int \int J_{s_1s_2}(t,\omega)dtd\omega}
\]

(9)

However, one must be careful in defining the information measure of the cross time-frequency distribution which is a complex number. Therefore, instead of direct application of the generalized Rényi information, consider the time-frequency coherence spectrum described above. The coherence spectrum is a measure of linearity between two time series, and we can define time-frequency coherence \( C_{s_1s_2}(t,\omega) \) as follows:

\[
C_{s_1s_2}(t,\omega) = \frac{J_{s_1s_2}(t,\omega)}{\sqrt{C_{s_1}(t,\omega) \cdot C_{s_2}(t,\omega)}}
\]

\[
= \Re\{\frac{J_{s_1s_2}(t,\omega)}{\sqrt{C_{s_1}(t,\omega) \cdot C_{s_2}(t,\omega)}}\} + j\Im\{\frac{J_{s_1s_2}(t,\omega)}{\sqrt{C_{s_1}(t,\omega) \cdot C_{s_2}(t,\omega)}}\}
\]

(10)

Therefore, we can define the time-frequency co-spectrum \( R_{s_1s_2}(t,\omega) \) and quad-spectrum \( Q_{s_1s_2}(t,\omega) \) as follows:

\[
R_{s_1s_2}(t,\omega) = \Re\{\frac{J_{s_1s_2}(t,\omega)}{\sqrt{C_{s_1}(t,\omega) \cdot C_{s_2}(t,\omega)}}\}
\]

\[
Q_{s_1s_2}(t,\omega) = \Im\{\frac{J_{s_1s_2}(t,\omega)}{\sqrt{C_{s_1}(t,\omega) \cdot C_{s_2}(t,\omega)}}\}
\]
We can define the information measure of the time-frequency co-spectrum \( I_{s_1,s_2} \) and quad-spectrum \( Q_{s_1,s_2} \) as follows:

\[
H_\alpha(R_{s_1,s_2}) = \frac{1}{1 - \alpha} \log_2 \int \int I_{s_1,s_2}^\alpha (t, \omega) dt d\omega - \frac{1}{1 - \alpha} \log_2 \int \int (\sqrt{C_{s_1}(t, \omega) \cdot C_{s_2}(t, \omega)})^\alpha dt d\omega
\]

\[
= \frac{1}{1 - \alpha} \log_2 \int \int I_{s_1,s_2}^\alpha (t, \omega) dt d\omega - \frac{1}{1 - \alpha} \frac{1}{2} \log_2 \int \int C_{s_1}^\alpha (t, \omega) dt d\omega + \log_2 \int \int C_{s_2}^\alpha (t, \omega) dt d\omega
\]

\[
= \frac{1}{1 - \alpha} \log_2 \int \int I_{s_1,s_2}^\alpha (t, \omega) dt d\omega - \frac{1}{2} \log_2 \{ H_\alpha(C_{s_1}) + H_\alpha(C_{s_2}) \}
\]

\[
H_\alpha(Q_{s_1,s_2}) = \frac{1}{1 - \alpha} \log_2 \int \int Q_{s_1,s_2}^\alpha (t, \omega) dt d\omega - \frac{1}{2} \log_2 \{ H_\alpha(C_{s_1}) + H_\alpha(C_{s_2}) \}
\]

Then, the mutual information measure \( I_\alpha(C_{s_1}; C_{s_2}) \) of \( S_1 \) and \( S_2 \) is defined in terms of co-spectral mutual information \( I_\alpha(R_{s_1,s_2}) \) and quad-spectral mutual information \( I_\alpha(Q_{s_1,s_2}) \) as follows:

\[
I_\alpha(C_{s_1}; C_{s_2}) = I_\alpha(R_{s_1,s_2}) + I_\alpha(Q_{s_1,s_2})
\]

\[
= - H_\alpha(I_{s_1,s_2}) - H_\alpha(Q_{s_1,s_2})
\]

\[
= - \frac{1}{1 - \alpha} \log_2 \int \int I_{s_1,s_2}^\alpha (t, \omega) dt d\omega + \log_2 \int \int Q_{s_1,s_2}^\alpha (t, \omega) dt d\omega + H_\alpha(C_{s_1}) + H_\alpha(C_{s_2})
\]

\[
= \frac{H_\alpha(C_{s_1}) - (H_\alpha(C_{s_1}; C_{s_2}) - H_\alpha(C_{s_2}))}{H_\alpha(C_{s_1}|C_{s_2})} = H_\alpha(C_{s_1}) - H_\alpha(C_{s_1}|C_{s_2})
\]

\[
= \frac{H_\alpha(C_{s_2}) - (H_\alpha(C_{s_1}; C_{s_2}) - H_\alpha(C_{s_1}))}{H_\alpha(C_{s_2}|C_{s_1})} = H_\alpha(C_{s_2}) - H_\alpha(C_{s_2}|C_{s_1})
\]

Therefore, the mutual time-frequency information \( I_\alpha(C_{s_1}; C_{s_2}) \) is the sum of individual time-frequency information \( H_\alpha(C_{s_1}), H_\alpha(C_{s_2}) \) and joint information \( H_\alpha(C_{s_1}, C_{s_2}) \). If \( s_1(t) = s_2(t) \), then \( C_{s_1} = C_{s_2} \) and \( Q_{s_1,s_2} = 0 \) such that

\[
I_\alpha(C_{s_1}; C_{s_2}) = I_\alpha(C_{s_1}; C_{s_2}) = H_\alpha(C_{s_1}) \quad \text{or} \quad H_\alpha(C_{s_2})
\]

Based on the mutual time-frequency information measure, we investigate the efficacy of the proposed technique with real-world data sets. The experimental setup and data descriptions are provided in the next section.

3. EXPERIMENTAL SETUP AND DATA DESCRIPTION

The CBM center at The University of South Carolina has an AH-64 Helicopter tail rotor driveshaft apparatus for collecting the data that was analyzed.\(^1\) The apparatus includes an electric motor to provide power to the system, a multi-shaft drive train supported by hanger bearings, two gearboxes, and an absorption motor to simulate the torque loads that would be applied by the tail rotor blade as shown in Figure 2(a). The apparatus was used to collect data to be used in conjunction with actual helicopter vibration data to develop the baseline of operation for the systems. The signals being collected during the operational running of the apparatus included vibration data measured by the accelerometers, temperature measured via thermocouples, and speed and torque measurements. The measurement devices were placed at the forward and aft hanger bearings and both gearboxes. This paper focuses on the application of time-frequency techniques to the forward and aft hanger bearing vibration signals denoted \( S_1 \) and \( S_2 \).

The data acquisition program collected data from the hanger bearings once every two minutes during the course of the thirty minute run, with the exception of two additional collection periods at the start of the run. The data format of the time series is provided in Figure 2(b). Each time that data was collected the memory buffer of the sensor was mixed at 65536 points. This collection was done with a sampling rate of 48 kHz which results in a data collection time of roughly 1.31 sec per acquisition. For each run, data was collected seventeen times: twice at the beginning and then once every two minutes until the end of the run. This results in over one million data points per set, which is too much for the processor to handle during time-frequency analysis. In order to resolve this computational issue, each data set was divided into 17 experiments to correspond to each time the sensor was activated to collect data. Each of the 17 experiments was then divided into 16 windows comprised of 4096 points each. As indicated in Figure 2(b), the time-frequency analysis was applied to these individual windows.
3.1. Power Spectral Density

For a preliminary study of the signals, the power spectral density is calculated for both the forward and aft hanger bearings for each of the two data sets (00321 and 20321). In Figure 3, power spectral density of $S_1$ and $S_2$ are provided. Figure 3(a) and Figure 3(b) are the power spectral densities of 00321 (baseline) and 20321 (misaligned).

From the power spectral densities provided in Figure 3, the dominant frequencies can be identified for each of the cases and bearings. For the 00321 case in Figure 3(a), the dominant frequencies for the forward hanger bearing ($S_1$)
include 82.2, 244.3, 487.8, 569.3, and 5049 Hz. The dominant frequencies for the aft hanger bearing ($S_2$) include 24.2, 82.2, 244.3, 488.6, 1590, and 2274 Hz. Using the common helicopter frequencies, it can be seen that the common frequencies between the two hanger bearings (82, 244 and 488 Hz) correspond to the tail rotor driveshaft frequency and its third and fifth harmonics. For the 20321 case in Figure 3(b), the dominant frequencies for the forward hanger bearing ($S_1$) include 82, 244, and 5075 Hz. The dominant frequencies for the aft hanger bearing ($S_2$) include 25, 244, and 1130 Hz. The common frequency between the two hanger bearings (244 Hz) corresponds to its third harmonic.

By comparing the power spectral densities in Figure 3(a) and Figure 3(b), some differences can be found to distinguish the misaligned shaft condition. The first and most noticeable is the drop in power seen in the two most dominant frequency terms (82 and 244 Hz). This power loss can be partially attributed to the spreading of some of the power among other frequencies in the system and to straight power loss as a result of the misaligned condition shown in Figure 3(b). Another indicator is the appearance of the third harmonic in the misaligned case. Its power level is low in the balanced case, but it becomes the most dominant component in the aft hanger bearing for the misaligned case. The last indicator of note is that the three most dominant frequencies for the balanced case are held in common for the two hanger bearings. In summary, we can obtain rough descriptions of the baseline and misaligned shafts from the classical power spectral density. However, it is not an easy task to determine the health status of the shaft by comparing the classical power spectral densities.

3.2. Analysis via Spectrogram

Time-frequency distribution in the form of spectrogram was used to identify time-frequency signatures of different experimental setups. In Figures 4 and 5, a set of spectrograms of $S_1$ and $S_2$ are provided for the baseline shaft and the misaligned shaft. The top portions of figures are time series, and the time-frequency distribution is provided in the same time axis.

The classical power spectral density results in Figure 3 cannot be clearly determined using the time-varying spectral characteristics of each signal set. It should be noted that the traditional energy spectral density plots look almost identical in the frequency domain, but the vibration signatures in the time-frequency domain exhibit distinctive characteristics. Instead, observing the time-frequency distributions provided in Figure 4, one can quantify physical parameters like instantaneous frequency and frequency bandwidth. From analysis of the spectrograms, the existence of the dominant frequencies seen from the power spectral density can be confirmed and can easily be seen as the high density stripes in the 3kHz and under range.

For example, two distinct frequency stripes can be seen in the aft hanger bearing readings. The higher of these frequency stripes (1.5-2 kHz range) is not found on the forward hanger bearing. This frequency stripe is attributed to frequencies emanating from the tail rotor gearbox. With relation to the forward hanger bearing signal, the distinguishing diagnostic indicators from the spectrogram reveal differences in the frequency stripes for the baseline and misaligned cases. The major differences in the forward hanger bearing reading between the balance and

![Figure 4. Spectrogram of $S_1$ and $S_2$ in 00321 (baseline).](image-url)
The number of signal elements on time-frequency plane can be mathematically assessed using the Rényi information measure. The Rényi information measures of time-frequency distribution in Figure 4(a) and Figure 4(b) are 6.83 and 6.71. One can find more time-frequency signal components in Figure 4(a) at 15-20 kHz frequency bandwidth, which results in a slightly higher value of Rényi information measure of the spectrogram in Figure 4(a). In addition, the Rényi information measures of time-frequency distribution in Figure 5(a) and Figure 5(b) are 7.83 and 6.93. Comparing the sets of spectrograms in Figure 4 and Figure 5 illustrates that the spectrograms in Figure 5 exhibit more time-frequency components than those in Figure 4, and one can quantitatively confirm a reasonable measure of time-frequency information using the Rényi information measure.

These differences and signatures on the time-frequency domain cannot be distinguished from the traditional power spectrum reading, a fact which is made apparent from the quantitative reading of the Rényi information. Further, the results are not sufficient to describe mutual interactions between $S_1$ and $S_2$ in different experimental setups. In the next section, we investigate the efficacy of time-frequency based mutual information measure in order to quantitatively characterize the experimental setups of the baseline and misaligned shafts.

4. RESULTS AND DISCUSSION

The first step of analysis and discussion uses the self Rényi information measure defined in (4) to describe the individual time series. The self Rényi information measures of $S_1$ and $S_2$ for baseline and misaligned cases are provided in Figure 6. Signal 1 and Signal 2 in both the 00321 and 20321 cases are analyzed by applying the 8-point moving average filtering followed by Rényi information calculation to obtain the self information measure described in Section 2.1. Thus, for every time instance of every experiment window of the data, a Rényi calculation of each auto-correlated signal was gathered. As shown in Figure 6, a total of 272 self information measures were gathered for each signal of each case. In order to identify the tendency of the measure, an 8-point moving average filter was applied to each signal with the filter covering half of the time instances provided in each experiment window. The results of this self information measure are compared side by side in Figure 6 for each signal. Time instance (15th of window number at 5th experiment) is marked on each graph to show consistency with the analyses in Section 2 and Section 3.2.

Notable from the side by side comparison in Figure 6 is a sizable increase in the self information measure of the misaligned case over the baseline case. This can be a characteristic signature of a misaligned case. This measure can be double-checked using the spectrogram example discussion in Figure 5. From this data, there is little other indication of change from the baseline case to another “faulty” status of the shaft. Moreover, the self Rényi information of $S_1$ in the balanced case in Figure 6(a), as well as both signals in the misaligned case, oscillate more compared to the self Rényi information of $S_2$ of the baseline case (00321). This could be attributed to more high frequency

Figure 5. Spectrogram of $S_1$ and $S_2$ in 20321 (misaligned).
Figure 6. Self Rényi information measure of $S_1$ and $S_2$ for baseline in (a)∼(b), and self Rényi information measure of $S_1$ and $S_2$ for misalignment in (c)∼(d)

components shown in the time-frequency spectrograms of Figures 4 and 5.

While this information proves useful and shows a notable basis by which to compare data sets, further information can be gathered from the mutual information measure. This mutual information measure is a complex value and can be further subdivided into two constituent values: a co-spectral mutual time-frequency information ($I_\alpha(R_{s_1 s_2})$) and a quad-spectral mutual time-frequency information ($I_\alpha(Q_{s_1 s_2})$). Mutual information measures of baseline and misaligned cases are provided in Figure 7. An interesting trend can be seen in the baseline case in Figure 7(a). Overall, the co-spectral mutual time-frequency information ($I_\alpha(R_{s_1 s_2})$) stays mostly at a constant separation from the quad-spectral mutual time-frequency information ($I_\alpha(Q_{s_1 s_2})$). Both the $I_\alpha(R_{s_1 s_2})$ and $I_\alpha(Q_{s_1 s_2})$ of the baseline case in Figure 7(a) remain relatively constant throughout all windows of the experiment. However, toward the end of the sequence outlined in Figure 7(a), the in-phase and quadrature mutual information measure values begin to experience a larger separation. These characteristics are all important to note while considering what truly characterizes the baseline physics of the system.

A glance at the mutual information from the misaligned case in Figure 7(b) draws attention to two distinctive signatures. First, like the baseline case, the co-spectral mutual time-frequency information ($I_\alpha(R_{s_1 s_2})$) remains relatively constant throughout all experiment windows with a large trough around experiment window 10 corresponding to a minimum value of the quad-spectral mutual time-frequency information ($I_\alpha(Q_{s_1 s_2})$). Second, the quadrature component has a larger average value over the length of the experiments than was seen in the quadrature component in the baseline case. Also, the quadrature component in the misaligned case fluctuates greatly, showing greater amounts of local minima and maxima. Though the quadrature information in the misaligned case revealed a significant rise in the number of bits in the mutual information measure, the in-phase portion showed little increase over the experiment windows measure. By comparing the results in Figure 7 with other results by classical spectral analysis or traditional spectrogram, one can find the usefulness of the proposed technique for a quantitative health condition assessment of the experimental setup.
Figure 7. Mutual information measures of baseline 00321 in (a) and misaligned 20321 in (b).

5. CONCLUSION

In summary, the baseline can be characterized with a constant separation on a per-time instance basis of the mutual information measure. The misaligned case may be characterized by its quadrature component. This component shows the misalignment in a relatively large, increased number of bits from the information measure. However, similarity still remains in the in phase component whether the case is aligned or misaligned. This would prove useful when other cases are considered and will be expounded upon in further research. The authors are interested in the fusion of other types of sensors in order to obtain extended information for more accurate assessment of the health status. Furthermore, analysis of these values can yield great insights into the physics behind systems such as that which provided the mechanical vibration data, providing either a simple summary of component health for an operator or a complex interpretation from a knowledgeable engineer in order to fully achieve condition-based maintenance.

ACKNOWLEDGMENTS

This research is funded by the South Carolina Army National Guard and United States Army Aviation and Missile Command via the Conditioned-Based Maintenance Research Center at the University of South Carolina-Columbia. Also, this research is partially supported by the National Science Foundation Faculty Early Career Development (CAREER) Program. The authors appreciate the NASA EPSCoR Undergraduate Scholarship for Mr. David Coats and the Republic of Korea Army Overseas Education Scholarship for Cpt. Kwangik Cho.
REFERENCES


